Technical Reports

CENTRO EURO-MEDITERRANEO
PER I CAMBIAMENTI CLIMATICI

ISC – Divisione Impatti sul Suolo e sulle Coste

Final document on the second year, second activity: "Preliminary results obtained by using the 2D numerical propagation model and sensitivity analysis with respect to the parameters"

Prof. Francesco Macchione*

Ing. Pierfranco Costabile*

Ing. Carmelina Costanzo*

* *LAMPIT (LAboratorio di Modellistica numerica per la Protezione Idraulica del Territorio) – Department of Soil Defence – University of Calabria*

**Centro Euro-Mediterraneo
per i Cambiamenti Climatici**
www.cmcc.it

June 2008 ■ TR

Final document on the second year second activity: "Preliminary results obtained by using the 2D numerical propagation model and sensitivity analysis with respect to the parameters"

Abstract

This report is the final document related to the second year, second activity whose title is: "Preliminary results obtained by using the 2D numerical propagation model and sensitivity analysis with respect to the parameters".
The purpose of the collaboration between LAMPIT (Department of Soil Defence, University of Calabria) and CMCC is to develop an hydrometeorological chain in order to obtain a reliable tool in the context of flood evolution prediction able to provide quantitative information of practical importance within the civil protection activities.
The LAMPIT contribution to the project concerns the mathematical description of both the generation and propagation of flood events at basin scale. The work here presented, carried out in close cooperation with dr. Pasquale Schiano and CIRA researchers (dr. Paola Mercogliano and dr. Gabriella Ceci), has been organised as follows:

- Analysis of the models proposed in the technical literature with particular reference to the kinematic and diffusive approximations.
- Collection and description of numerical tests usually used in the scientific context for the validation of the numerical models.
- Development and implementation of 2D models.
- Validation of the models developed by LAMPIT through numerical tests.

Keywords: Hydrometeorological chains, Flood propagation

JEL Classification:

Address for correspondence:
Prof. Francesco Macchione
Laboratorio LAMPIT
Dipartimento di Difesa del Suolo – Università della Calabria
Ponte P.Bucci – Cubo 42/B
87036, Rende (CS) .
E-mail: f.macchione@unical.it

PRELIMINARY REMARKS

Flood numerical modelling: basic aspects

The analysis of flood events due to heavy rainfall needs a strict and innovative methodology capable to provide the evolution of the phenomena in relation with potential extreme events as well as their spatial and temporal distribution within a selected area.
In order to obtain a reliable prediction of the hydraulic risk associated to extreme events, the use of numerical simulation models, appropriately validated using both experimental and real event data, seems to be necessary. Such models are able to carry out a quantitative evaluation of the most important parameters in the context of the hydraulic protection of the territory as water depths, velocities and flooded areas.
Starting from the knowledge accumulated over the years within the LAMPIT laboratory (Department of Difesa del Suolo, University of Calabria) on the performances of several finite volume schemes for the numerical integration of the shallow water equations, two codes have been implemented: the HLL first order of accuracy upwind scheme and the MacCormack second order of accuracy central scheme. The previous choice is consequent to a large comparative survey on the performances of several first and second order upwind and central finite volume numerical schemes as HLL, HLLC, Roe scheme, MacCormack-TVD scheme (Costanzo & Macchione, 2004; Costanzo & Macchione, 2005a; Costanzo & Macchione, 2005b; Costanzo & Macchione, 2006). That analysis has been carried out focusing the attention on both computational aspects, such as the implementation burdensomeness and computational times, and engineeristic aspects as the accuracy of the solution in terms of maximum water levels, arrival times and velocities. From the above mentioned papers, it may be deduced that the simulations carried out by means of the MacCormack scheme are the most accurate predictions; the HLL scheme also works very well and it is very competitive in terms of computational times.

Simplification of the shallow water equations

The availability of high resolution topographic data may induce the development of several novelties (newnesses) in the context of the flood modeling (Macchione 2008). Actually, the LIDAR (*LIght Detection and RAnging*) technology provides a very detailed cartographic data since it is possible to obtain a vertical and horizontal accuracy of 10^{-1} m and 2 m respectively. Therefore the computational domain may be generated using a very small grid size. The high resolution simulation of flooding events seems thus to be very feasible. However, both the computational costs and memory storage capacity of the standard computers represent a significant limit for achieving that goal.
On the other hand, as already stated in the first year (second technical report), the convective inertial terms in the momentum equations are significantly lower than the values of the topographic surface slope in those situations in which a strong altimetrical gradient occurs; as a consequence, the aforementioned terms may be ignored.
The simplification of the governing equations seems thus to be natural in order to avoid an useless increase of the computational times in the propagation model due to the large extension of the areas that have to be considered in the real cases.
For that reason, the momentum equation is often proposed in a simplified form according to both the kinematic and diffusive approximations; for example, Natale & Savi (1991), Defina et al., (1994), Molinaro et al. (1994), Tucciarelli et al. (1995) have proposed numerical models neglecting the inertial convective terms in the momentum equation.
Conceptual approaches, known as quasi-2D models, have been also presented in order to simplify the governing equations. Such models are based on a flooded area representation by means of cells hydraulically connected with the neighbouring cells. The first model of this category have

been proposed by Zanobetti & Lorgeré (1968) and by Zanobetti et al. (1970); other schemes have been presented also by Balloffet & Scheffler (1982), Laura & Wang (1984), Maione et al. (1986), Reitano (1992).
The Hromadka & Yen (1986) model is based on a 1D approach for channel flow and a 2D representation of the flooded areas. Once again the inertial terms are neglected obtaining a diffusive scheme; the connection between the river flow and 2D cells is possible by means of continuity equation.
Bladé et al. (1994) proposed a model in which the channel flow is simulated with a 1D approach while the flooding areas processes are described by a number of cells hydraulically connected by weir-type flow or uniform flow laws. It was assumed also that the flow discharge may be expressed as a function of the water surface levels that are the system variables. The boundary conditions may be introduced by imposing water levels or external discharges.
With reference to urban flooding, Natale (1988), Braschi & Gallati (1989) have proposed numerical schemes according to which the urban area is schematized as a series of nodes, connected by channels, in which a storage volume occurs. The flooded areas are evaluated considering the density of the buildings in the selected zone. In a similar way, Molinaro et al. (1994) take into account the urban area by introducing, for every cells, coefficients less than one computed as the ratio between the flooded area and total cell area. Frega et al. (1999) propose a model based on a triangular cell representation of the territory hydraulically connected by weir type relationships; for each cell the continuity equation is applied. The model has just one calibration parameter and it was able to numerically reproduce the flooding event of Crotone city (14 october 1996) in a good way.
More recently, Yu & Lane (2006a) have analysed the performance of a model based on the continuity equation together with the Manning law in order to describe the discharge flow between neighbouring cells. The numerical results show that the model is very sensitive to both cell size and Manning coefficients. In order to overcome the previous problems, Yu & Lane (2006b) have proposed an approach based on the availability of high resolution topographic data that provide a better representation of both the storage volumes and the flow transfer mechanisms. A more accurate description of the phenomenon may be obtained by the connection with a 1D unsteady flow model for the river flow (Yu & Lane, 2007).
A similar model has been proposed by Bates & De Roo (2000) and successively developed by Hunter et al. (2005). That model is based on the 1D kinematic approximation for the channel flow while the flooding processes are described using the continuity equations and the Manning law for each computational cell; the diffusive version of the previous model have been presented by Horritt & Bates (2001).
Even the dam break flood wave, in particular conditions it is possible also to simplify the flow equations. Macchione & Viggiani (2000) have analysed those criteria proposed in the literature according to which the approximations of the shallow water equations are justified and the results obtained with a simplified diffusive-type model (Macchione 1994) are compared with a fully unsteady flow model.

Simplification of the shallow water equations in the context of the overland flow modeling

With reference to the problem of the generation and propagation of a flood wave at basin scale, the use of different approximations of the unsteady flow equations is very common in order to simulate the overland flow processes. Several authors have studied the conditions in which that approximations are completely justified. The analysis carried out by Woolhiser (1974) is presented herein.

Starting from the unsteady flow equations written using as variables the water depth h and the velocity U:

$$\frac{\partial h}{\partial t}+\frac{\partial}{\partial x}(Uh)=q(x,t)$$
$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+g\frac{\partial h}{\partial x}=g\left(S_0-S_f\right)-q(x,t)\frac{U}{h}$$

in which $q(x,t)$ is the net rain, S_0 is the surface slope e S_f friction slope, with reference to a inclined impervious plane of unit width, length L_0 and assuming that the rainfall rate is not a function of x or t, the above equations may be rewritten in a nondimensional forms as follows:

$$\frac{\partial h^*}{\partial t^*}+\frac{\partial}{\partial x^*}\left(U^*h^*\right)=1$$
$$\frac{\partial U^*}{\partial t^*}+U^*\frac{\partial U^*}{\partial x^*}+\frac{1}{F_0^2}\frac{\partial h^*}{\partial x^*}=\frac{S_0L_0}{H_0F_0^2}\left(1-\frac{U^{*2}}{h^*}\right)-\frac{U^*}{h^*}$$

in which:

$$h^*=\frac{h}{H_0};\;\; U^*=\frac{U}{U_0};\;\; x^*=\frac{x}{L_0};\;\; t^*=\frac{U_0t}{L_0};\;\; F_0=\frac{U_0}{\sqrt{gh_0}};\;\; q^*=\frac{q}{q_0}=1$$

H_0 is the normal depth at x=L_0 corresponding to a flow rate of $(q\,L_0)$ and $U_0=\frac{qL_0}{H_0}$.

Woolhiser & Liggett (1967) showed that, with reference to an inclined plane, as the kinematic wave number $K_0=\frac{S_0L_0}{H_0F_0^2}$ increases, the solution from the kinematic wave model approaches the solutions for the full Saint Venant equations; in particular, for K_0>20 e F_0>0.5, the kinematic wave model performs well and it is a good approximation to the Saint Venant equations of overland flow.

Morris & Woolhiser (1980) claimed that for highly subcritical flows (F_0<0.5) and when the upstream boundary condition is an important factor ($K_0F_0^2<5$), the kinematic wave fails and the corresponding hydrographs receded much faster than those for the Saint Venant equations. Such evenience does not occur when one uses a model based on the diffusion wave model. It is important to observe that the kinematic model is also incapable of incorporating backwater effects (Woolhiser 1974).

In the recent literature, there are a number of papers that propose the analysis of the overland flow problem by means of simplified models applied to both ideal case and small real basin. The model that LAMPIT will use in the project clearly depends also from the state of the art on that argument. In this context, the work carried out during the second year, second activity, has been organized according the following steps:

- Analysis of the models proposed in the technical literature with particular reference to the kinematic and diffusive approximations.
- Collection and description of numerical tests usually used in the scientific context for the validation of the numerical models.
- Development and implementation of 2D models.

- Validation of the models developed by LAMPIT (Department of Soil Defence, University of Calabria) through numerical tests.

ANALYSIS OF THE MODELS PROPOSED IN THE TECHNICAL LITERATURE WITH PARTICULAR REFERENCE TO THE KINEMATIC AND DIFFUSIVE APPROXIMATIONS

The main purpose of this working phase was to make a critical review of the approaches proposed in the literature in order to define a number of features of the model. An accurate analysis of the models proposed in the literature has been organised according to three focal points: the governing equations as well as the numerical integration schemes used, suggestions on the initial, boundary conditions and the stability criteria and information on the numerical applications carried out by the authors. The above survey represent the reference context for developing and implementing the numerical models showed in the next sections. The results of the analysis are now presented.

Howes, D.A., Abrahams, A. D., Pitman, E. B. (2006). "One- and two-dimensional modelling of overland flow in semiarid shrubland, Jornada basin, New Mexico". Hydrological processes, Wiley, 20: 1027.1046.

Main purpose of the paper

The authors propose two overland flow models based on the kinematic wave approximation according to both a 1D and 2D approach.

Governing equation and numerical integration scheme

The governing equations of the model are:

$$\frac{\partial h}{\partial t}+\frac{\partial q}{\partial x}+\frac{\partial q}{\partial y}=r-f \tag{1}$$

$$q=vh \tag{2}$$

The velocity v is computed according to the kinematic wave approximation; therefore, the flow velocity is calculated using an uniform flow relationship and in particular the Darcy-Weisbach law written in the form $q=\alpha h^m$ where:

$$\alpha=\sqrt{\frac{8gi}{f}}\ ;\ m{=}1.5$$

in which i is the surface slope and f is the friction factor.

A simplified version of the numerical model proposed by Davis (1988) is used; the Davis algorithm is an explicit second-order accurate predictor-corrector finite difference scheme with shock capturing properties; the second order accuracy in the spatial dimensions is obtained using center-differencing, by using the midpoint rule in which the water depth h for a cell is computed in two steps: at Δt/2, the midpoint of a given time step, first (predictor step) and at the end of the time step then (corrector step).

In particular, by substituting the equation (2) in the relation (1) the method may be summarized as follows:

$$h_{i,j}^{t+\Delta t} = h_{i,j}^{t} - v_x \frac{\Delta t}{2\Delta x}\left(h_{i+1,j}^{t+\Delta t/2} - h_{i-1,j}^{t+\Delta t/2}\right) - v_y \frac{\Delta t}{2\Delta y}\left(h_{i,j+1}^{t+\Delta t/2} - h_{i,j-1}^{t+\Delta t/2}\right) \tag{3}$$

where v_x and v_y are the flow velocities along the two directions.
However, it is not clear whether v_x and v_y are also evaluated at $t+\Delta t$ in the equation (3).

Specific suggestions on the initial and boundary conditions and stability criterion

The computation of Δt is carried out by using two criteria, both of which must be satisfied. The first of these is the Courant condition written in the following form:

$$\Delta t \le C \frac{\Delta x}{v} \tag{4}$$

The value of the Courant number C used by the authors is less than 0.1.
The second stability constraint applies to source term. If A is the magnitude of the source term, then the numerical scheme will be stable as long as:

$$\Delta t \le \frac{0.1}{A} \tag{5}$$

No theoretical explanations or references are highlighted to justify the above criterion.

Numerical applications

The model has been used to the analysis of the flood evolution over two small basins (700-900 m^2) discretized according to a $1m^2$ computational grid size. The maximum values of rainfall intensities are approximately 100-150 mm/h.

Tsai, T.-L., Yang, J.-C. (2005).”Kinematic wave modelling of overland flow using characteristics method with cubic spline interpolation”. Advances in Water Resources, Elsevier, 28: 661-670.

Main purpose of the paper

The authors show the performance of the CSMOC (Characteristics method with cubic spline interpolation) scheme proposed for the integration of the kinematic wave model and carried out a comparative analysis with other numerical scheme such as Preissmann model.

Governing equation and numerical integration scheme

By substituting in the 1D version of equation (1) the relationship (2) written in the form:

$$q = \alpha h^{\beta} \tag{6}$$

It is possible to obtain the following expression:

$$\frac{\partial h}{\partial t}+\alpha\beta h^{\beta-1}\frac{\partial h}{\partial x}=e\left(x,t\right) \tag{7}$$

where $e(x,t)$ is the net rain.
The previous equation may be rearranged as follows:

$$\frac{Dh}{Dt}=e\left(x,t\right) \tag{8}$$

along

$$\frac{dx}{dt}=\alpha\beta h^{\beta-1} \tag{9}$$

in which:

$$\frac{Dh}{Dt}=\left(\frac{\partial}{\partial t}\right)+\left(\frac{dx}{dt}\right)\left(\frac{\partial}{\partial x}\right) \tag{10}$$

denotes the total derivative.
The integration of equation (8) along the characteristic line (9) is carried out by means of the trapezoidal rule approximation leading to the following algebric relationship:

$$\begin{aligned} h_p-h_l&=\frac{\Delta t}{2}\left(e_p-e_l\right)\\ x_p-x_l&=\frac{\alpha\beta\Delta t}{2}\left(h_p^{\beta-1}+h_l^{\beta-1}\right)\end{aligned} \tag{8a,9a}$$

in which l and p are two nodal points of the characteristics trajectory; h_p is the unknown water depth at time level n while h_l is the water depth at time level (n-1) that may be computed by interpolation using the known grid values at the same time; the *cubic-spline* interpolation is the method proposed by the authors in this papers. That approach is used also to deal with 2D problems.

Specific suggestions on the initial and boundary conditions and stability criterion
No specific suggestions.

Numerical applications
The authors use their model to simulate ideal phenomenon such as a variable in time heavy rain, in a constant slope (4%) channel.

Moramarco, T., Singh, V. P. (2002). "Accuracy of kinematic wave and diffusion wave for spatial-varying rainfall excess over a plane". Hydrological Processes, Wiley, 16: 3419:3435.

Main purpose of the paper

The authors evaluate the accuracy achieved by using both the kinematic and diffusive wave in those situation in which the boundary conditions effects cannot be ignored.

Governing equation and numerical integration scheme

The governing equations are written in the following form:

$$\frac{\partial \mathbf{f}}{\partial t}+\frac{\partial \mathbf{p}(\mathbf{f})}{\partial x}=\mathbf{S} \tag{11}$$

According to the diffusive approximation the equation (11) reads:

$$\mathbf{f}=\begin{bmatrix} h \\ 0 \end{bmatrix};\quad \mathbf{p}(\mathbf{f})=\begin{bmatrix} uh \\ gh \end{bmatrix};\quad \mathbf{S}=\begin{bmatrix} i \\ g\left(S_0-S_f\right) \end{bmatrix} \tag{12,13,14}$$

while introducing the kinematic wave model one has:

$$\mathbf{f}=\begin{bmatrix} h \\ 0 \end{bmatrix};\quad \mathbf{p}(\mathbf{f})=\begin{bmatrix} uh \\ 0 \end{bmatrix};\quad \mathbf{S}=\begin{bmatrix} i \\ g\left(S_0-S_f\right) \end{bmatrix} \tag{15,16,17}$$

In both cases, the system (1) is solved by using the Lax-Wendroff scheme:

$$\begin{aligned} \mathbf{f}_{j+1/2}^{n+1/2} &= \frac{\mathbf{f}_j^n+\mathbf{f}_{j+1}^n}{2}-\frac{\Delta t}{2\Delta x}\left(\mathbf{p}_{j+1}^n-\mathbf{p}_j^n\right)+\frac{\Delta t}{4}\left(\mathbf{S}_{j+1}^n-\mathbf{S}_j^n\right) \\ \mathbf{f}_j^{n+1} &= \mathbf{f}_j^n-\frac{\Delta t}{\Delta x}\left(\mathbf{p}_{j+1/2}^{n+1/2}-\mathbf{p}_{j-1/2}^{n+1/2}\right)+\frac{\Delta t}{2}\left(\mathbf{S}_{j+1/2}^{n+1/2}+\mathbf{S}_{j-1/2}^{n+1/2}\right) \end{aligned} \tag{18,19}$$

Specific suggestions on the initial and boundary conditions and stability criterion:

Diffusive wave approximation:

upstream:

$$\begin{aligned} &u(0,t)=0 \\ &h_1^{n+1}=h_2^{n+1}-\left[S_0-\frac{S_f}{2}\bigg|_{j=2}^{n}\right]\Delta x \end{aligned} \tag{20}$$

downstream:

- critical depth $$h_N^{n+1} = h_{N-1}^{n+1} + \left[S_0 - \frac{S_{f_{N-1}} + S_{f_N}}{2} \right]^n \Delta x \quad (21)$$

- zero depth gradient $$u_N^{n+1} = K \left(h_{N-1}^{n+1} \right)^{2/3} \sqrt{S_0} \quad (22)$$

Kinematic wave approximation:

upstream:

$$h(0,t)=0$$

downstream (characteristic lines): $$\begin{aligned} \frac{dh}{dt} &= e(x,t) \\ \frac{dx}{dt} &= \beta\alpha h^{\beta-1} \end{aligned} \quad (23)$$

in which β,α are the same coefficients used in the equation (6).

Numerical applications

The authors apply their models to ideal situation concerning a time variable rain over a plane.

Kazezyilmaz-Alhan, C., Medina, M. A. (2007). "Kinematic and diffusion waves: analytical and numerical solutions to overland and channel flow". Journal of Hydraulic Engineering, ASCE, 133(2): 217-228.

Main purpose of the paper

In this paper the author:

1. show an analytical solution to the kinematic and diffusive wave;
2. propose the application of MacCormack scheme to the overland and channel flow problem;
3. analyse the accuracy of the scheme with reference to a real case concerning a very small basin (0.22 km^2)

Governing equation and numerical integration scheme

The governing equations are formulated for a kinematic and diffusive model according to a 1D approach in the following form:

Kinematic: $$\begin{cases} \dfrac{\partial h}{\partial t} + \dfrac{\partial (uh)}{\partial x} = e \\ u = \alpha h^\beta \end{cases} \quad (24,25)$$

Diffusive:
$$\begin{cases} \dfrac{\partial h}{\partial t} + u\dfrac{\partial h}{\partial x} + h\dfrac{\partial u}{\partial x} = e \\ u = \dfrac{1}{n} h^{2/3} \sqrt{S_0 - \dfrac{\partial h}{\partial x}} \end{cases} \qquad (26,27)$$

Deriving the equation (27) with respect to x one has:

$$\frac{\partial u}{\partial x} = \frac{2}{3}\frac{u}{h}\frac{\partial h}{\partial x} - \frac{1}{2}\frac{u}{S_0 - \dfrac{\partial h}{\partial x}}\frac{\partial^2 h}{\partial x^2} \qquad (28)$$

By substituting equation (28) in the (26) one may obtain:

$$\frac{\partial h}{\partial t} + C\frac{\partial h}{\partial x} = D\frac{\partial^2 h}{\partial x^2} + e \qquad (29)$$

where

$$C = \frac{5}{3}u; \quad D = \frac{hu}{2\left(S_0 - \dfrac{\partial h}{\partial x}\right)} = \frac{hu}{2S_f} \qquad (30,31)$$

The authors propose the solution of equation (29) by means of MacCormack scheme in which the second derivative, at the right hand side, is computed through the standard approximation:

$$\frac{\partial^2 h}{\partial x^2} \approx \frac{\left(h_{j+1} - 2h_j + h_{j-1}\right)^n}{\Delta x^2} \qquad (32)$$

in particular, the predictor and corrector steps, written using the discharge Q and cross-sectional area A, are expressed as follows:

1. kinematic wave

$$\overline{A_i^{j+1}} = A_i^j - \alpha\frac{\Delta t}{\Delta x}\left[\left(A_{i+1}^j\right)^m - \left(A_i^j\right)^m\right] \quad \text{predictor}$$

$$A_i^{j+1} = \frac{1}{2}\left\{A_i^j + \overline{A_i^{j+1}} - \alpha\frac{\Delta t}{\Delta x}\left[\left(\overline{A_i^{j+1}}\right)^m - \left(\overline{A_{i-1}^j}\right)^m\right]\right\} \quad \text{corrector}$$

2. diffusive wave

$$\overline{Q_i^{j+1}} = Q_i^j - c_i^j \frac{\Delta t}{\Delta x}\left[Q_{i+1}^j - Q_i^j\right] + \left(K_1\right)_i^j \frac{\Delta t}{\left(\Delta x\right)^2}\left(Q_{i+1}^j - 2Q_i^j + Q_{i-1}^j\right) \quad \text{predictor}$$

$$Q_i^{j+1} = \frac{1}{2}\left[\left(Q_i^j + \overline{Q_i^{j+1}}\right) - \overline{c_i^{j+1}} \frac{\Delta t}{\Delta x}\left[\overline{Q_i^{j+1}} - \overline{Q_{i-1}^{j+1}}\right] + \overline{\left(K_1\right)_i^{j+1}} \frac{\Delta t}{\left(\Delta x\right)^2}\left(\overline{Q_{i+1}^{j+1}} - 2\overline{Q_i^{j+1}} + \overline{Q_{i-1}^{j+1}}\right)\right] \quad \text{corrector}$$

Specific suggestions on the initial and boundary conditions and stability criterion

No particular suggestion but the Courant number values used by the authors ranging between 0.045 and 0.2

Numerical applications:

The model is used to simulate ideal situation concerning a heavy rain (both constant and variable in time) over a plane. Moreover a simulation over a experimental basin located in the Duke University Campus (Durham, NC, USA) is presented; the computation have been only carried out with 1D approach.

Gottardi, G., Venutelli, M. (2008). "An accurate time integration method for simplified overland flow models". Advances in Water Resources, Elsevier, 31: 173-180.

Main purpose of the paper

The authors propose a temporal integration method for the kinematic and diffusive models; that approach is based on a spatial discretization according to Lax-Wendroff scheme; the temporal solution is obtained by an analytical integration.

Governing equation and numerical integration scheme

The authors propose 1D and 2D models based on both a kinematic and a diffusive approximation. The 2D unsteady flow equations are

$$\begin{aligned}
&\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = e(x,y,t)\\
&\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + g\frac{\partial h}{\partial x} = g\left(S_{0x} - S_{fx}\right) - \frac{e}{h}\left(u - u'\right)\\
&\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial y} + v\frac{\partial v}{\partial y} + g\frac{\partial h}{\partial y} = g\left(S_{0y} - S_{fy}\right) - \frac{e}{h}\left(v - v'\right)
\end{aligned} \qquad (33,34,35)$$

in which u' e v' are the component of the mean flow velocity due to lateral inflow. By ignoring the latter terms together with inertial terms, the equations (34) and (35) assume the following form:

$$\frac{\partial h}{\partial x} = S_{0x} - S_{fx}$$
$$\frac{\partial h}{\partial y} = S_{0y} - S_{fy} \qquad (36,37)$$

The flow velocity component u and v are evaluate according the relationship proposed by Hromadka & Lai (1985):

$$u = Kh^m \frac{S_{fx}}{\sqrt{S_f}}; \quad v = Kh^m \frac{S_{fy}}{\sqrt{S_f}} \qquad (38,39)$$

where:

$$S_f = \sqrt{S_{fx}^2 + S_{fy}^2} \qquad (40)$$

Deriving u and v, with respect to x and y, respectively, and substituting in the continuity equation it is possible to obtain the following relationship:

$$\frac{\partial h}{\partial t} + C_x \frac{\partial h}{\partial x} + C_y \frac{\partial h}{\partial y} = D_x \frac{\partial^2 h}{\partial x^2} + D_y \frac{\partial^2 h}{\partial y^2} + e(x, y, t) \qquad (41)$$

where:

$$C_x = (m+1)u; \quad C_y = (m+1)v \qquad (42,43)$$

$$D_x = \frac{uh}{2S_{fx}}\left[1 + \left(\frac{S_{fy}}{S_f}\right)^2\right]; \; D_y = \frac{vh}{2S_{fy}}\left[1 + \left(\frac{S_{fx}}{S_f}\right)^2\right] \qquad (44,45)$$

The 2D kinematic model may be obtained from equation (41) neglecting the diffusive terms:

$$\frac{\partial h}{\partial t} + C_x \frac{\partial h}{\partial x} + C_y \frac{\partial h}{\partial y} = e(x, y, t) \qquad (46)$$

In the equation (46) the celerities are computed according to the kinematic approximation in the following way:

$$u = Kh^m \frac{S_{0x}}{\sqrt{S_0}}; \quad v = Kh^m \frac{S_{oy}}{\sqrt{S_0}} \qquad (47,48)$$

where:

$$S_0 = \sqrt{S_{0x}^2 + S_{0y}^2} \tag{49}$$

The numerical model is only briefly described for the 1D approach by the author.
The second derivatives are computed according to relationship (32), while the spatial derivatives are evaluated by using the Lax-Wendroff scheme. Therefore, the numerical integration scheme may be written in the following form:

$$\frac{\partial h_j}{\partial t} = \frac{D_j}{(\Delta x)^2}\left(h_{j-1} - 2h_j + h_{j+1}\right) - \frac{C_j}{2\Delta x}\left[\left(h_{j+1} - h_{j-1}\right) - \upsilon\left(h_{j-1} - 2h_j + h_{j+1}\right)\right] + e_j \tag{50}$$

where υ is the Courant number. The temporal integration is carried out in a exact manner once a Taylor expansion series of the terms C_j and D_j is performed. In summary, the scheme may be expressed in the following form:

$$h_j^{n+1} = h_j^n + \frac{\frac{D_j}{(\Delta x)^2}\left(h_{j-1} - 2h_j + h_{j+1}\right) - \frac{C_j}{2\Delta x}\left[\left(h_{j+1} - h_{j-1}\right) - \upsilon\left(h_{j-1} - 2h_j + h_{j+1}\right)\right] + e_j}{B}\left(e^{B\Delta t} - 1\right) \tag{51}$$

with:

$$B = \frac{\left(h_{j-1} - 2h_j + h_{j+1}\right)}{(\Delta x)^2}\left.\frac{dD}{dh_j}\right|_{h_j^n} - \frac{\left(h_{j-1} - 2h_j + h_{j+1}\right)}{(\Delta x)^2}\left.\frac{dC}{dh_j}\right|_{h_j^n} \tag{52}$$

Specific suggestions on the initial and boundary conditions and stability criterion

The authors report a discussion on the boundary conditions, briefly reassumed herein.
The zero inflow upstream boundary condition is often taken into account considering:

$$u(0,t) = 0$$

which in the case of kinematic model leads to:

$$h(0,t) = 0$$

while for the diffusive model it reads:

$$\frac{\partial h}{\partial x}(0,t) = S_0$$

The previous upstream boundary conditions are not compatible with the dry bed initial condition $h(x,0) = 0$; therefore the condition $h(0,t) = 0$ must be used also for the diffusive model.

The downstream boundary, critical flow or zero depth gradient boundary conditions are suggested. The critical flow condition may be expressed by the relationship $u(L,t)=\sqrt{gh(L,t)}$ that for the diffusive model reads:

$$h(L,t)^{m-\frac{1}{2}}\left(S_0-\frac{\partial h(L,t)}{\partial x}\right)^{\frac{1}{2}}=\frac{\sqrt{g}}{K}$$

while the zero depth gradient boundary condition leads to:

$$\frac{\partial h}{\partial x}(L,t)=0$$

Little differences between the hydrographs obtained by using the previous conditions were found by Govindaraju et al. (1988).

Numerical applications

The authors applied the model to simulate ideal situation concerning a heavy rain, both constant and variable in time, over a plane or surface with variable slope. No real world application are shown.

Feng, K., Molz, G. J. (1997). "A 2-D, diffusion-based, wetland flow model". Journal of Hydrology, Elsevier, 196: 230-250.

Main purpose of the paper

The authors propose a 2D diffusive model whose governing equations are formulated using the surface water level H.

Governing equation and numerical integration scheme

The equations of the model are:

$$\begin{aligned}&\frac{\partial H}{\partial t}+\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}=0\\&S_{fx}=-\frac{\partial H}{\partial x}\\&S_{fy}=-\frac{\partial H}{\partial y}\end{aligned}\qquad(53,54,55)$$

In their introduction, the authors illustrate the importance of an accurate evaluation of Manning coefficient. Moreover, with reference to a study carried out by Turner and Chanmeesri (1984), they highlight a resistance formula more consistent with the concept of "diffusion" than the Manning law. That formula, made explicit according to the discharge q, may be expressed as follows:

$$q = \frac{h^{\alpha} S_f^{\gamma}}{G} = -\frac{h^{\alpha}}{G}\left(\frac{\partial H}{\partial s}\right)^{\gamma} \tag{56}$$

where α,γ e G are parameters.
Then it is possible obtain the following relationship:

$$q_x = q\cos(\theta); \quad q_y = q\sin(\theta) \tag{57,58}$$

$$\frac{\partial H}{\partial x} = \frac{\partial H}{\partial s}\cos(\theta); \quad \frac{\partial H}{\partial y} = \frac{\partial H}{\partial s}\sin(\theta); \tag{59,60}$$

in which θ is the angle between the flow direction and the axis x.
After some algebraic manipulations one may obtain:

$$q_x = -\frac{h^{\alpha}}{G}\left|\frac{\partial H}{\partial s}\right|^{\gamma-1}\left(\frac{\partial H}{\partial s}\right)\cos(\theta) = -\frac{h^{\alpha}}{G}\left|\frac{\partial H}{\partial s}\right|^{\gamma-1}\frac{\partial H}{\partial x} = -\frac{h^{\alpha}}{G\left|\frac{\partial H}{\partial s}\right|^{1-\gamma}}\frac{\partial H}{\partial x} \tag{61}$$

and in a similar way:

$$q_y = -\frac{h^{\alpha}}{G\left|\frac{\partial H}{\partial s}\right|^{1-\gamma}}\frac{\partial H}{\partial y} \tag{62}$$

Combining the equation (53) with the relationships (61) and (62) and using h=H-z, with z bed surface level, one has the following expression:

$$\frac{\partial H}{\partial t} = \frac{\partial}{\partial x}\left(\frac{(H-z)^{\alpha}}{G\left|\frac{\partial H}{\partial s}\right|^{1-\gamma}}\frac{\partial H}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{(H-z)^{\alpha}}{G\left|\frac{\partial H}{\partial s}\right|^{1-\gamma}}\frac{\partial H}{\partial y}\right) \tag{63}$$

in which the term $\frac{\partial H}{\partial s}$ is evaluable starting from equation (59) or (60) observing that:

$$\theta = \tan^{-1}\left(\frac{\partial H/\partial y}{\partial H/\partial x}\right) \tag{64}$$

Consequently the equation (63) may be rewritten as follows:

$$\frac{\partial H}{\partial t}=\frac{\partial}{\partial x}\left(D_x\frac{\partial H}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_y\frac{\partial H}{\partial y}\right) \tag{65}$$

where:

$$D_x=\frac{(H-z)^\alpha}{G\left|\frac{\partial H}{\partial s}\right|^{1-\gamma}};\qquad D_y=\frac{(H-z)^\alpha}{G\left|\frac{\partial H}{\partial s}\right|^{1-\gamma}} \tag{66}$$

The equation (65) is solved by means of an implicit finite difference method using the Picard-iteration scheme for the non-linear terms (Ames 1992) not illustrated herein.
It has to be said that the authors in their numerical application use the Manning equation instead of equation (56) due to the absence of experimental evaluation.

Specific suggestions on the initial and boundary conditions and stability criterion

The authors present a particular treatment aimed at the automatic determination of the no flow boundary front location by means of the diffusion coefficient *D*.
The no-flow boundary front moves when the water surface elevation changes. With reference to figure 1, at time t_1 the water surface at node (i) is lower than the land surface at (i+1) so between (i) and (i+1) the diffusion coefficient D=0 and the node (i) is the no-flow boundary front. At time t_2, the water surface at node (i+1) is lower than the land surface at (i+2), between (i+1) and (i+2) the diffusion coefficient D=0 and the no-flow boundary front moves to node (i+1). By specifying the value of D in this way, the moving no flow boundary front can be determined automatically during the solution of the process.

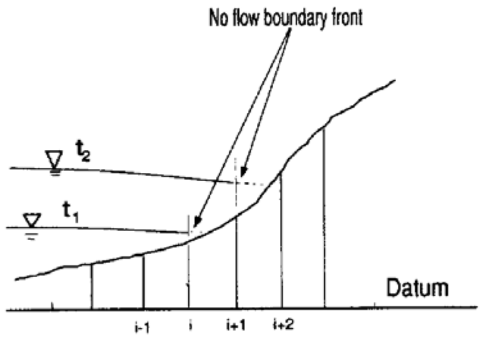

Figure 1. Logic scheme of the procedure aimed at the automatic location of no flow boundary front. (by Feng & Molz 1997)

Numerical application

The authors use their model to numerically reproduce three experimental tests concerning a spatially variable rain (but constant in time) over a variable slope surface; they consider also a real event over a small basin (13000 m^2) discretized with 1m^2 cells.

Di Giammarco, P., Todini, E., Lamberti, P. (1996). "A conservative finite elements approach to overland flow: the control volume finite element formulation". Journal of Hydrology, Elsevier, 175: 276.291.

Main purpose of the paper

The authors propose a locally conservative formulation of the finite elements approach to the numerical integration of the 2D diffusion overland flow equations.

Governing equation and numerical integration scheme

The authors propose a diffusive model based on the equations (53,54,55). The friction slope are written in the following way:

$$S_{fx} = \frac{n_x^2}{h^{4/3}} |\mathbf{w}| \mathbf{w} \times \mathbf{i} = \frac{n_x^2}{h^{4/3}} \sqrt{\left(u^2+v^2\right)} u; \quad S_{fx} = \frac{n_x^2}{h^{4/3}} |\mathbf{w}| \mathbf{w} \times \mathbf{j} = \frac{n_y^2}{h^{4/3}} \sqrt{\left(u^2+v^2\right)} v \qquad (66,67)$$

in which $\mathbf{w} = \mathbf{u_i} + \mathbf{v_j}$ is the velocity vector and therefore:

$$|w|^2 = u^2 + v^2 = h^{4/3} \left(\frac{S_{fx}^2}{n_x^4} + \frac{S_{fy}^2}{n_y^4} \right)^{\frac{1}{2}} \qquad (68)$$

So combining the equations (54) and (55) with (66) and (67) it is possible to make explicit the two components of the vector **w**:

$$u = -\frac{\partial H}{\partial x} \frac{h^{2/3}}{n_x^2} \frac{1}{\left[\left(\frac{\partial H}{\partial x} \right)^2 \frac{1}{n_x^4} + \left(\frac{\partial H}{\partial y} \right)^2 \frac{1}{n_y^4} \right]^{1/4}}; \quad v = -\frac{\partial H}{\partial y} \frac{h^{2/3}}{n_y^2} \frac{1}{\left[\left(\frac{\partial H}{\partial x} \right)^2 \frac{1}{n_x^4} + \left(\frac{\partial H}{\partial y} \right)^2 \frac{1}{n_y^4} \right]^{1/4}} \qquad (69,70)$$

By substituting the previous relationships in the equations (53) it is possible to obtain the following differential diffusive equation:

$$\frac{\partial H}{\partial t} = \frac{\partial}{\partial x}\left(k_x \frac{\partial H}{\partial x} \right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial H}{\partial y} \right) + e(x,y,t) \qquad (71)$$

where:

$$k_x = \frac{1}{n_x^2} \frac{h^{5/3}}{\gamma(\nabla H)}; \quad k_x = \frac{1}{n_y^2} \frac{h^{5/3}}{\gamma(\nabla H)} \qquad (72)$$

and

$$\gamma(\nabla H) = \left[\left(\frac{\partial H}{\partial x} \right)^2 \frac{1}{n_x^4} + \left(\frac{\partial H}{\partial y} \right)^2 \frac{1}{n_y^4} \right]^{1/4} \tag{73}$$

The numerical integration of equation (71) is carried out by the authors by means of a locally conservative finite element method known as CVFE (control volume finite element).

Specific suggestions on the initial and boundary conditions and stability criterion

No particular suggestions.

Numerical applications

The model has been applied to an ideal test for which it is possible to obtain an analytical solution. The test geometrical configuration is made of two hypothetical impervious hillsides with constant and transverse constant slope at whose bottom a constant slope channel is located. The rainfall is assumed also constant and it only falls on the hillsides.

COLLECTION AND DESCRIPTION OF NUMERICAL TESTS USUALLY USED IN THE SCIENTIFIC CONTEXT FOR THE VALIDATION OF THE NUMERICAL MODELS

This working phase is motivated by the need of providing a concise document concerning the numerical tests normally used in the technical literature to validate the overland flow models.
Clearly, the aim of this analysis is selecting the most significant situations to test the numerical results obtained by means of the models developed during the second year within LAMPIT laboratory (Department of Difesa del Suolo, University of Calabria).
The numerical tests have been classified according four categories:

- tests concerning a constant in space and time rainfall intensity over a plane;
- tests concerning a constant in space but variable in time rainfall intensity over a plane;
- tests concerning a constant in time but variable in space rainfall intensity over a cascade of plane
- tests concerning constant in time rainfall intensity over an ideal basin composed by two constant slope hillsides at whose bottom a constant slope channel is located.

Tests concerning a constant in space and time rainfall intensity over a plane

Test 1: *Rainfall intensity constant in time and space (0.33 mm/min), duration 200 min over a plane 400 m long, with constant slope (0.0005) and Manning coefficient n=0.02 s/m$^{1/3}$* (Figure 2).

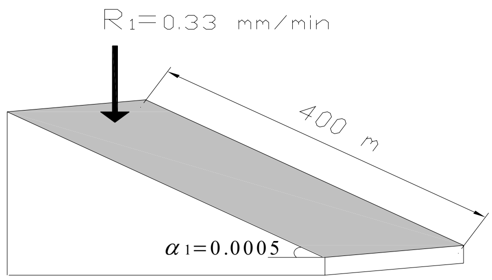

Figure 2 –Geometric configuration and rainfall data for test 1

Authors: Gottardi & Venutelli 1993; Jaber & Mohtar 2003

Comparison: unit discharge hydrographs (analytical solution – Figures 3 - 4)

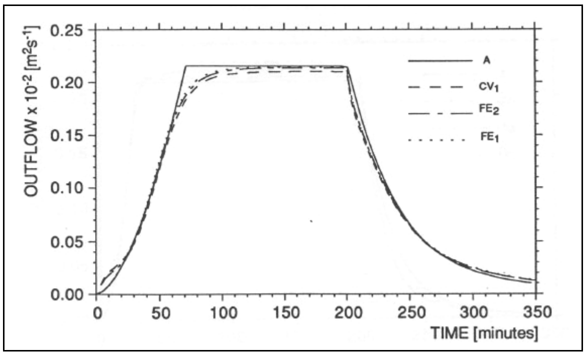

Figure 3 – Discharge outflow hydrographs for test 1: analytical solution and numerical simulation (from Gottardi & Venutelli 1993)

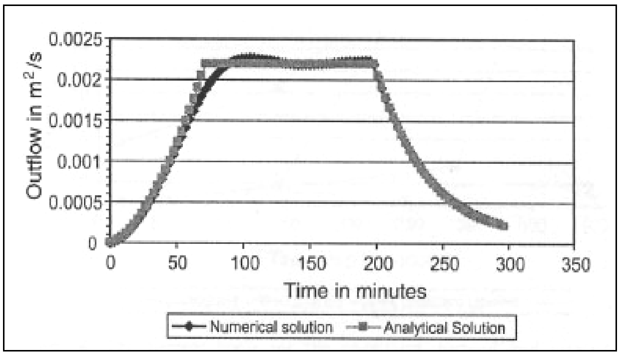

Figure 4 – Discharge outflow hydrographs for test 1: analytical solution and numerical simulation (from Jaber & Mohtar 2003)

Test 2: *Rainfall intensity constant in time and space (30 cm/h), duration 1600s over a plane, 1000m long, with constant slope (0.01) and Manning coefficient $n=0.02\ s/m^{1/3}$* (Fig. 5).

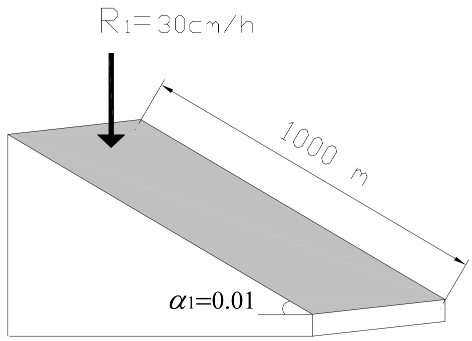

Figure 5 –Geometric configuration and rainfall data for test 2

<u>Authors</u>: Singh 1996, Tsai & Yang 2005

<u>Comparison</u>: water depths hydrographs in two sections located at 500m and 1000m far from the upstream boundary (analytical solution – Fig. 6)

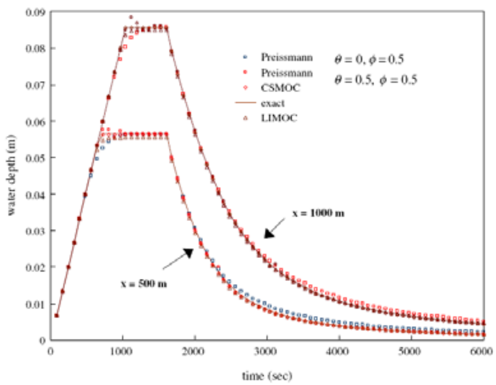

Figure 6 - Water depths hydrographs in two sections of the plane: analytical and numerical solution (from Tsai & Yang 2005)

Test 3: *Rainfall intensity constant in time and space (0.33 mm/h), duration 1h, over a plane, 200m long, with constant slope (0.001) and Manning coefficient n=0.03 s/m$^{1/3}$* (Figure 7).

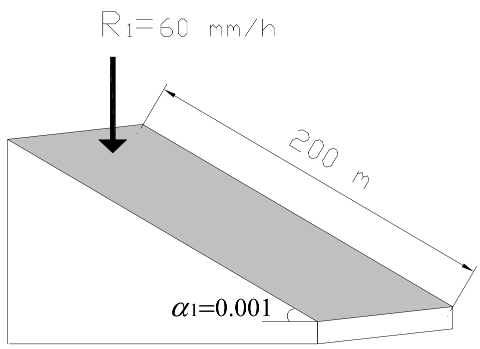

Figure 7 –Geometric configuration and rainfall data for test 3

<u>Autori</u>: Gottardi & Venutelli 2008

Confronto: runoff hydrographs computed as the unit discharge over the plane length (analytical solution – Figure 8)

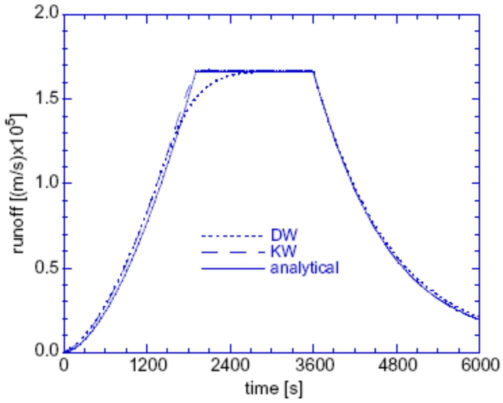

Figure 8 – Runoff hydrographs relative to test 3: analytical solution and numerical simulation (from Gottardi & Venutelli 2008)

Test 4a,4b: *Rainfall intensity constant in time and space* (92.96 mm/h; 46.48 mm/h *), "infinite" duration, over a plane, 20m long, with constant slope (0.001) and Manning coefficient n*=0.5 s/m$^{1/3}$ (0.4 s/m$^{1/3}$) (Figure 9)

Authors: Gottardi & Venutelli 2008

Comparison: runoff hydrographs computed as the unit discharge over the plane length (analytical solution, experimental data, numerical simulation, Figure 10)

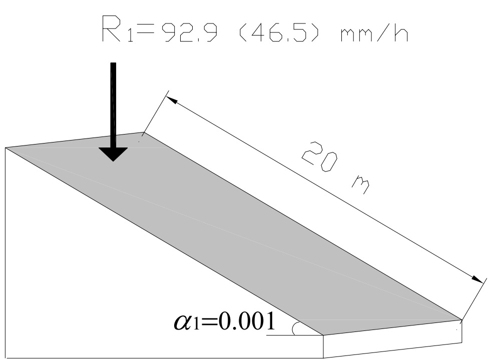

Figure 9 – Geometric configuration and rainfall data for test 4a,4b

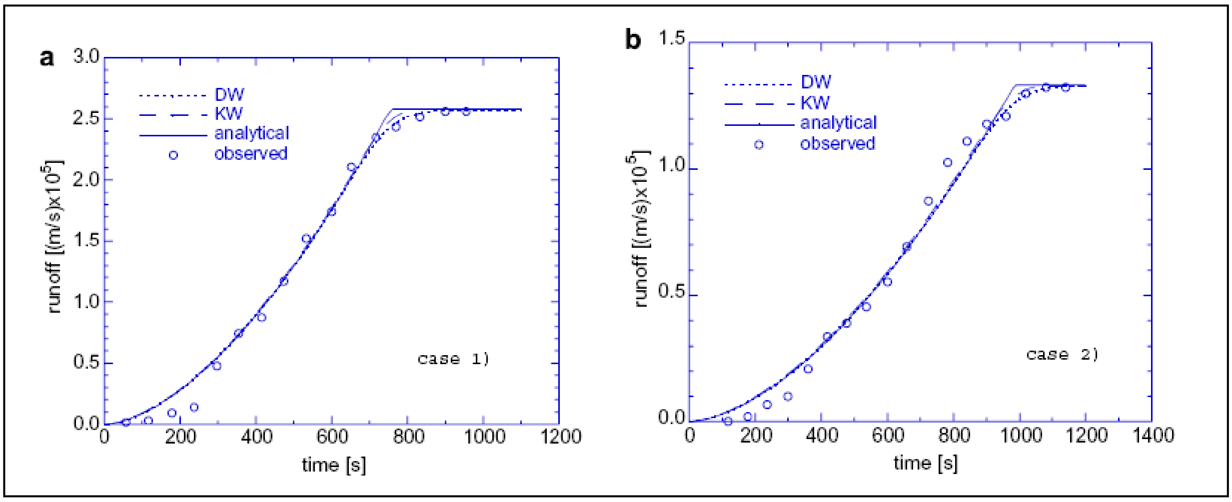

Figure 10 – Runoff hydrographs relative to test 4a,4b: analytical solution, experimental data, numerical simulation (from Gottardi & Venutelli 2008)

Tests concerning a constant in space but variable in time rainfall intensity over a plane

Test 5: *Variable in time rainfall intensity (table 1) over a plane, 1000m long, with constant slope (0.01) and Manning coefficient n=0.02 s/m$^{1/3}$ (Figure 11)*

Time (s)	0<t≤1500	1500t≤3000	3000<t≤4500	t>4500
Rainfall intensity (mm/h)	100	300	100	0

Table 1 - Rainfall intensity data for test 5

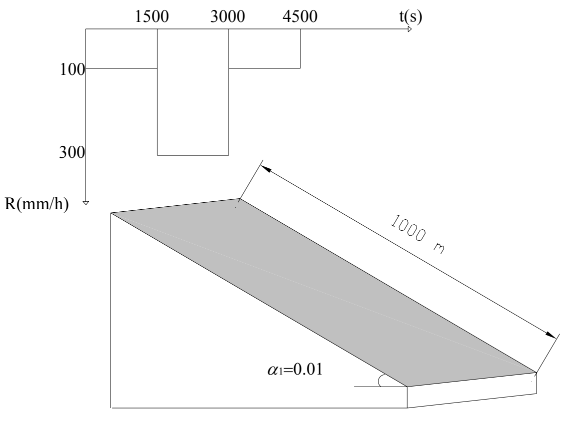

Figure 11 – Geometric configuration and rainfall data for test 5

Authors: Tsai & Yang 2005

Comparison: water depths hydrographs at the outlet of the plane (exact solution interpreted as the solution of the full unsteady flow equations by means of a finite element method – Figure 12)

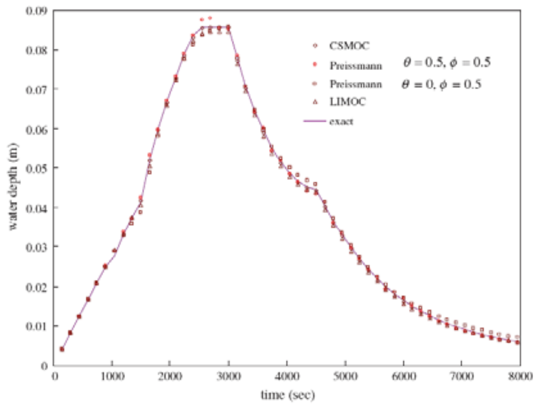

Figure 12 –Water depths at the plane outlet concerning test 5: numerical simulations (from Tsai & Yang 2005)

Test 6a: *Variable in time rainfall intensity (table 2) over a plane, 22m long, with constant slope (0.001) and Chézy coefficient* χ=1.336 $m^{1/2}$/s (Figure 13).

Time (s)	0<t≤600	600t≤1200	1200<t≤1800	1800<t≤2400	t>2400
Rainfall intensity (cm/h)	101.6	50.8	101.6	50.8	0

Table 2 – Rainfall data for test 6a

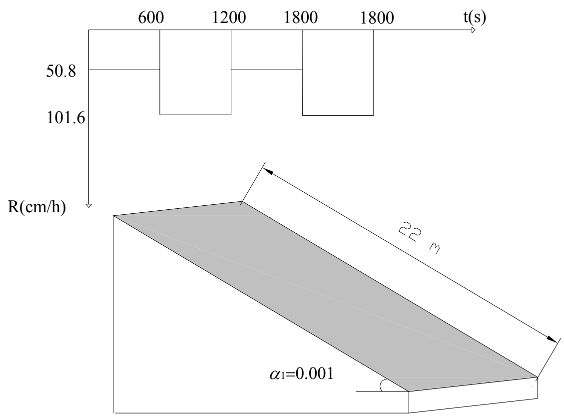

Figura 13 – Geometric configuration and rainfall data for test 6a

Authors: Govindaraju et al. 1988, Gottardi & Venutelli 2008

Comparison: runoff hydrographs computed as the unit discharge over the plane length (solution of the full unsteady flow equations – Figure 14)

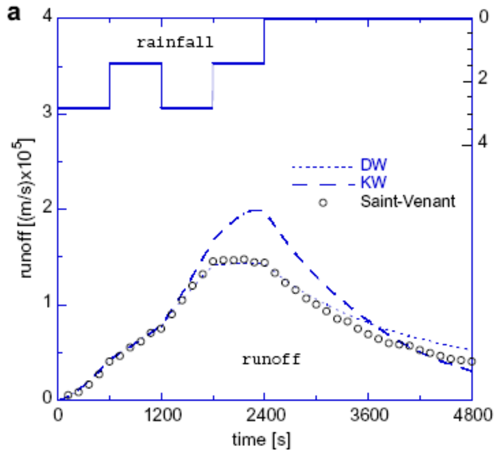

Figure 14 – Runoff hydrographs for test 6a: numerical simulation (from Gottardi & Venutelli 2008)

Test 6b: *Variable in time rainfall intensity (table 3) over a plane, 22m long, with constant slope (0.04) and Chézy coefficient* χ=1.336 $m^{1/2}/s$ (Figure 15).

Time (s)	0<t≤600	600t≤1200	1200<t≤1800	1800<t≤2400	t>2400
Rainfall intensity (cm/h)	50.8	101.6	50.8	101.6	0

Table 3 - Rainfall data for test 6b

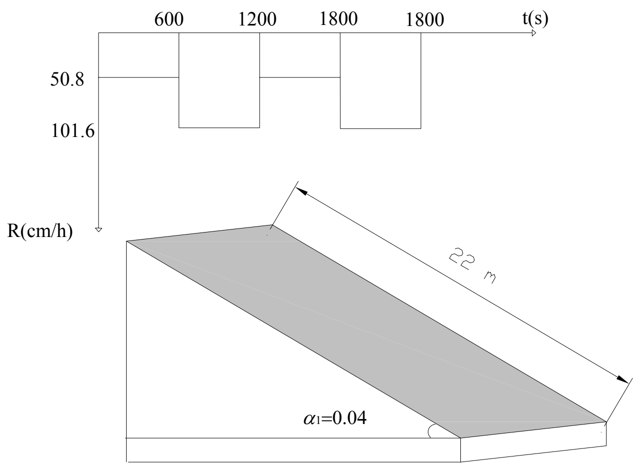

Figure 15 – Geometric configuration and rainfall data for 6b

Authors: Govindaraju et al. 1988, Gottardi & Venutelli 2008

Comparison: runoff hydrographs computed as the unit discharge over the plane length (solution of the full unsteady flow equations – Figure 16)

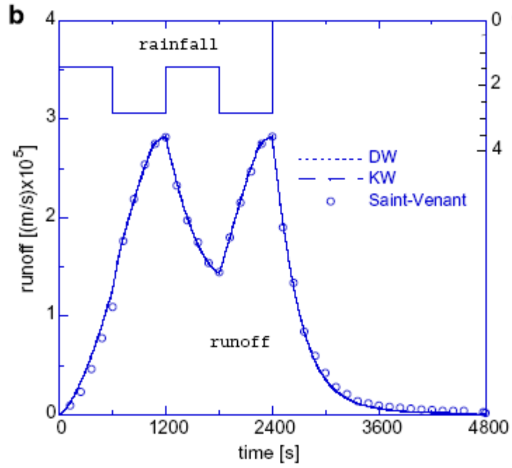

Figure 16 – Runoff hydrographs for test 6b: numerical simulation (from Gottardi & Venutelli 2008)

Test 6c: *Variable in time rainfall intensity (table 4) over a plane, 22m long, with constant slope (0.04) and Chézy coefficient* χ=1.767 m$^{1/2}$/s.

Time (s)	0<t≤600	600t≤1200	1200<t≤1800	1800<t≤2400	t>2400
Rainfall intensity (cm/h)	101.6	50.8	101.6	50.8	0

Table 4 - Rainfall data for test 6c

Authors: Govindaraju et al. 1988, Gottardi & Venutelli 2008

Comparison: runoff hydrographs computed as the unit discharge over the plane length (solution of the full unsteady flow equations – Figure 17)

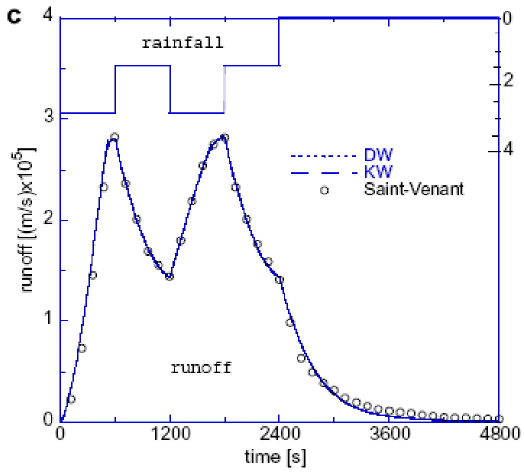

Figure 17 - Runoff hydrographs for test 6c: numerical simulation (from Gottardi & Venutelli 2008)

Tests concerning a constant in time but variable in space rainfall intensity over a cascade of planes

Test 7a,b,c: *Variable in space rainfall intensity (table 5) but constant in time over a cascade of planes, 24 long, with Manning coefficient n=0.009-0.01 s/m$^{1/3}$ (Figure 18).*

Distance from the upstream boundary (m)	$0<x\leq 8$	$8<x\leq 16$	$16<x\leq 24$
Slope	0.02	0.015	0.01
Rainfall intensity (cm/h)	389	230	288

Table 5 - Rainfall data for test 7

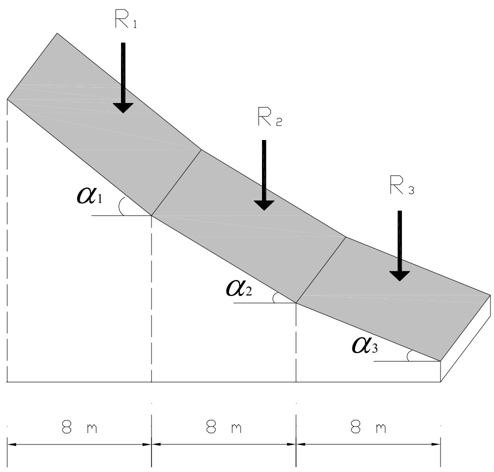

Figure 18 – Geometric configuration and rainfall data for tests 7a,7b and 7c.

Authors: Iwagaki 1955, Feng & Molz 1997, Fiedler & Ramirez 2000

Confronto: discharge hydrographs at the end of the experimental domain (Figures 19,20,21) and water depths profiles (Figure 22) obtained from different rainfall durations (experimental data)

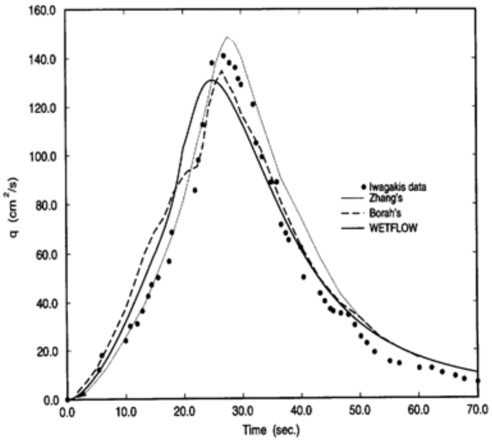

Figure 19 – Flood wave at the channel outlet: experimental points and numerical simulations. Rain duration t=30s (from Feng & Molz 1997)

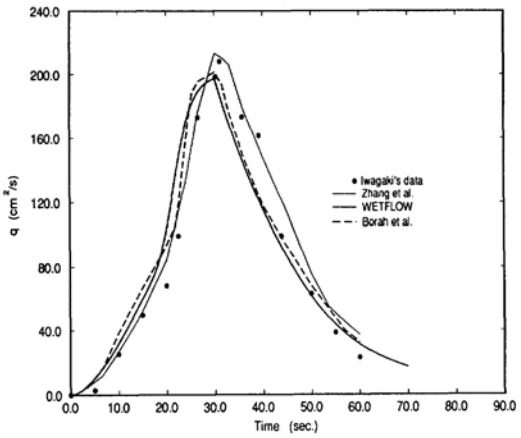

Figure 20 – Flood wave at the channel outlet: experimental points and numerical simulations. Rain duration t=20s (from Feng & Molz 1997)

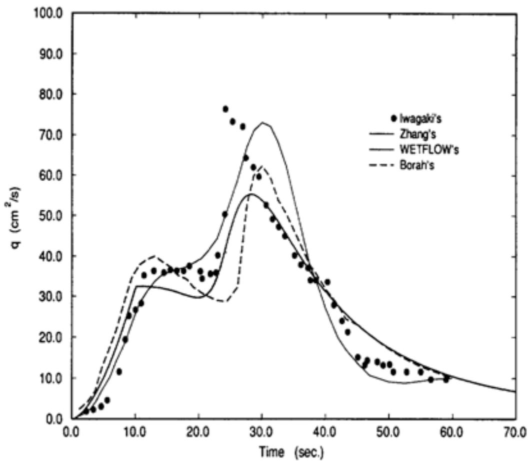

Figure 21 – Flood wave at the channel outlet: experimental points and numerical simulations. Rain duration t=10s (from Feng & Molz 1997)

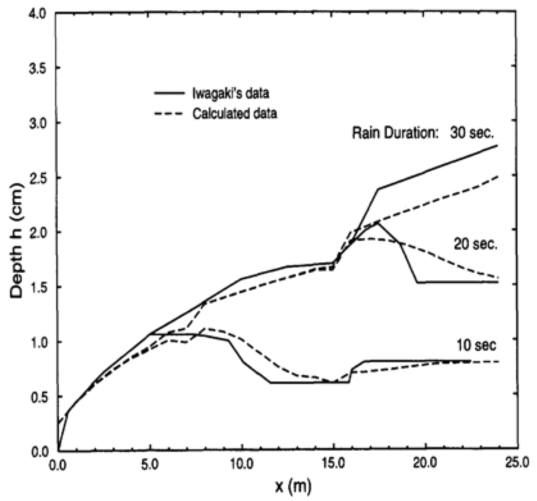

Figure 22 Longitudinal water depths profile at the end of the rainfall input in the three cases (from Feng & Molz 1997)

Tests concerning constant in time rainfall intensity over an ideal basin composed by two constant slope hillsides at whose bottom a constant slope channel is located.

Test 8: *Constant rainfall intensity over two plane 800x1000m, having Manning coefficient n=0.015 s/m$^{1/3}$, transversal slope 0.05 and no longitudinal slope, whose discharges flows into a constant slope (0.02) channel with Manning coefficient n=0.15 s/m$^{1/3}$(Figure 23).*

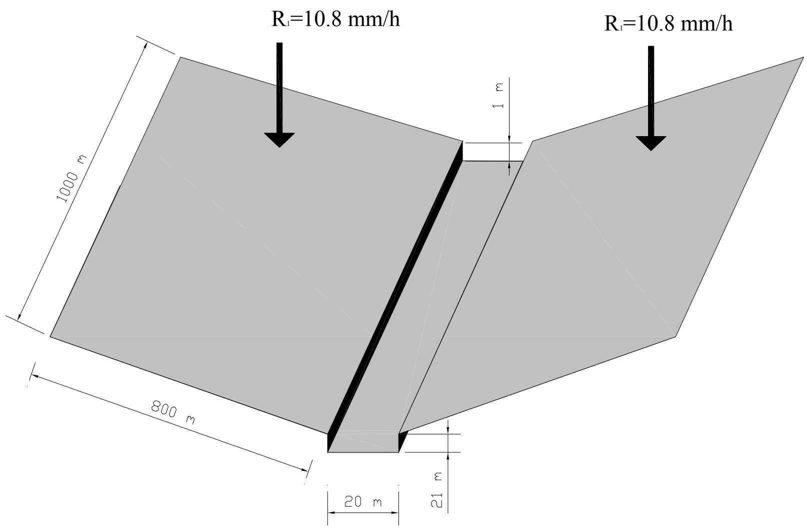

Figure 23 – Geometric configuration and rainfall data for test 8

Authors: Stephenson & Meadows 1986, Di Giammarco et al. 1996

Comparison: discharge hydrographs both at the bottom of the plane and at the channel outlet (Figures 24,25) (analytical solution)

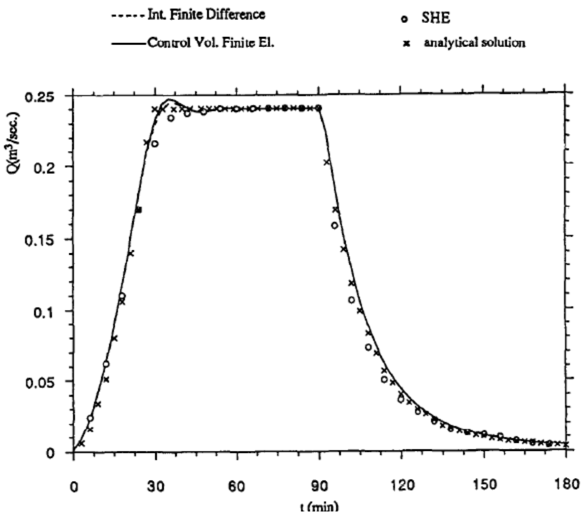

Figure 24 – Flood wave at the bottom of the plane

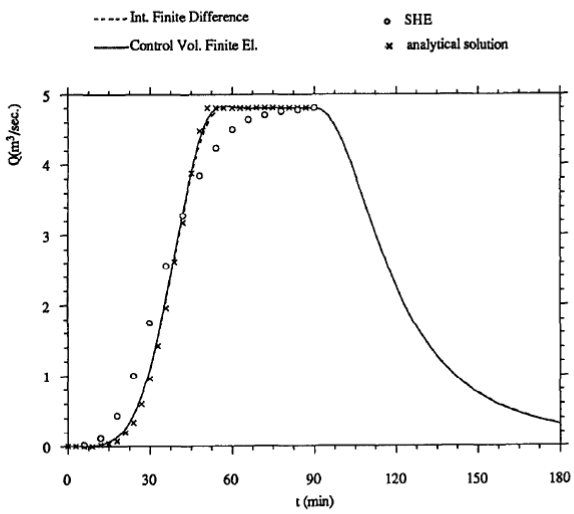

Figure 25 – Flood wave at the channel outlet

DEVELOPMENT AND IMPLEMENTATION OF TWO DIMENSIONAL MODELS

According to the previous bibliographical review, a number of two dimensional schemes have been implemented and validated in LAMPIT laboratory (Department of Soil Defence, University of Calabria) to the analysis of overland flow events.
The implemented codes are based on the fully conservative shallow water equations:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{S} \tag{74}$$

where:

$$\mathbf{U} = \begin{pmatrix} h \\ hu \\ hv \end{pmatrix} \tag{75}$$

$$\mathbf{F} = \begin{pmatrix} hu \\ hu^2 + gh^2/2 \\ huv \end{pmatrix};\ \mathbf{G} = \begin{pmatrix} hv \\ huv \\ hv^2 + gh^2/2 \end{pmatrix} \tag{76 , 77}$$

$$\mathbf{S} = \begin{pmatrix} r - f \\ gh\left(S_{0x} - S_{fx}\right) \\ gh\left(S_{0y} - S_{fy}\right) \end{pmatrix} \tag{78}$$

where:
t is time; x, y are the horizontal coordinates; h is the water depth; u, v are the depth-averaged flow velocity in x- and y- directions; g is the gravitational acceleration; S_{0x}, S_{0y} are the bed slopes in x- and y- directions; S_{fx}, S_{fy} are the friction slopes in x- and y- directions, which can be calculated from Strickler's formula; r is the rain intensity and f are the infiltration losses.
As already mentioned, in the simulation of overland flow events the convective inertial terms in the momentum equations are significantly lower than the values of the topographic surface slope in those situation in which a strong altimetrical gradient occurs. Whenever that eventuality happens, it is justified to neglect these terms in order to avoid an useless increase of the computational times in the propagation model due to the large extension of the areas that have to be considered in the real cases.
By neglecting the local and convective acceleration in the momentum conservation equations, it is possible to obtain the following diffusive model:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{S} \tag{79}$$

with:

$$\mathbf{U} = \begin{pmatrix} h \\ 0 \\ 0 \end{pmatrix} \tag{80}$$

$$\mathbf{F} = \begin{pmatrix} hu \\ gh^2/2 \\ 0 \end{pmatrix};\ \mathbf{G} = \begin{pmatrix} hv \\ 0 \\ gh^2/2 \end{pmatrix} \tag{81, 82}$$

$$\mathbf{S} = \begin{pmatrix} r - f \\ gh\left(S_{0x} - S_{fx}\right) \\ gh\left(S_{0y} - S_{fy}\right) \end{pmatrix} \tag{83}$$

and ignoring also the depth gradient terms one may obtain the following kinematic model:

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{S} \tag{84}$$

with:

$$\mathbf{U} = \begin{pmatrix} h \\ 0 \\ 0 \end{pmatrix} \tag{85}$$

$$\mathbf{F} = \begin{pmatrix} hu \\ 0 \\ 0 \end{pmatrix}; \ \mathbf{G} = \begin{pmatrix} hv \\ 0 \\ 0 \end{pmatrix} \tag{86 , 87}$$

$$\mathbf{S} = \begin{pmatrix} r - f \\ gh\left(S_{0x} - S_{fx}\right) \\ gh\left(S_{0y} - S_{fy}\right) \end{pmatrix} \tag{88}$$

Therefore, according to the characteristics of the flow, the complete model, as represented by the equations 74 -78, the diffusive model (equations 79-83) or the kinematic model (equations 84 - 88) may be used.

The finite volume method, widely adopted in the literature, has been used to discretize the previous equations. It considers the integral form of the shallow water equations that allows a quite easy implementation of shock capturing schemes on different mesh type.

The system of equations is integrated over an arbitrary control volume $\Omega_{i,j}$ and, in order to obtain surface integrals, the Green theorem has been applied to each component of the vectors **F** and **G** leading to:

$$\frac{\partial}{\partial t}\int_{\Omega_{i,j}} \mathbf{U} d\Omega + \oint_{\partial\Omega_{i,j}} [\mathbf{F},\mathbf{G}]\cdot\mathbf{n}\, dL = \int_{\Omega_{i,j}} \mathbf{S} d\Omega \tag{89}$$

where $\partial\Omega_{i,j}$ being the boundary enclosing $\Omega_{i,j}$, **n** is the unit vector normal and dL is the length of each boundary. Denoting by $\mathbf{U}_{i,j}$ the average value of the flow variables over the control volume $\Omega_{i,j}$ at a given time, the equation (89) may be discretized as:

$$\mathbf{U}_{i,j}^{n+1} = \mathbf{U}_{i,j}^{n} - \frac{\Delta t}{\Omega_{i,j}} \sum_{r=1}^{4} \left[\mathbf{F},\mathbf{G}\right]_r \cdot \mathbf{n}_r \, \Delta L_r + \Delta t \mathbf{S}_{i,j}^{n} \tag{90}$$

The finite volume method, as represented by the equation (90), allows the decomposition of a two dimensional problem into a series of locally one dimensional problems to value the normal flux through every side of a cell. Many algorithms have been proposed for the flux vectors evaluation: the most diffused approches proposed in the literature have been examined and implemented herein.

Generally the most popular finite volume schemes are *upwind* schemes and *central* schemes. In the upwind schemes the computational cells are selected according to the propagation of the perturbations while the central schemes are characterized by a central discretization of the flux vectors through a side of the cell.

In particular in the analysis presented herein the HLL first order upwind scheme has been implemented and used for integrating the complete and kinematic model while MacCormack second order space centered scheme has been applied to the complete, diffusive and kinematic model.

Several authors have used the MacCormack scheme to simulate the propagation of overland flow processes; among them Fiedler & Ramirez (2000); Gandolfi &Savi (2000); Esteves et al. (2000); Kazezyilmaz-Alhan &Medina (2007); Liang et al. (2007).
The HLL scheme only considers the left and right wave characteristics as representative of the minimum and the maximum speed of the perturbation which divide x^n.- t plain in three regions.
That scheme, applied to the two dimensional equations, gives the following expression for the numerical flux across the edge of the computational cell Ω_L on the left and Ω_R on the right:

$$[\mathbf{F},\mathbf{G}]_r \cdot \mathbf{n}_r = \begin{cases} [\mathbf{F},\mathbf{G}]_L \cdot \mathbf{n}_r & if \;\; s_L \geq 0 \\ [\mathbf{F},\mathbf{G}]^* \cdot \mathbf{n}_r & if \;\; s_L \leq 0 \leq s_R \\ [\mathbf{F},\mathbf{G}]_R \cdot \mathbf{n}_r & if \;\; s_R \leq 0 \end{cases} \tag{91}$$

where:

$$[\mathbf{F},\mathbf{G}]^* \cdot \mathbf{n}_r = \frac{s_R([\mathbf{F},\mathbf{G}])_L \cdot \mathbf{n}_r - s_L([\mathbf{F},\mathbf{G}])_R \cdot \mathbf{n}_r + s_L s_R(\mathbf{U}_R - \mathbf{U}_L)}{s_R - s_L} \tag{92}$$

The expressions of the wave celerities are:

$$s_L = \min\left([u,v]_L \cdot \mathbf{n}_r - \sqrt{gh_L}, u^* - \sqrt{gh^*}\right) \tag{93}$$

$$s_R = \max\left([u,v]_R \cdot \mathbf{n}_r + \sqrt{gh_R}, u^* + \sqrt{gh^*}\right) \tag{94}$$

with:

$$u^* = \frac{1}{2}([u,v]_L + [u,v]_R) \cdot \mathbf{n}_r + \sqrt{gh_L} - \sqrt{gh_R} \tag{95}$$

$$\sqrt{gh^*} = \frac{1}{2}\left(\sqrt{gh_L} + \sqrt{gh_R}\right) + \frac{1}{4}([u,v]_L - [u,v]_R) \cdot \mathbf{n}_r \tag{96}$$

In the case of the discretization of the kinematic model, the equations (91 -96) have been only applied to the mass conservation equation while the momentum equations, along the two directions x and y, have been simply resolved computing the velocities through the kinematic equations as Gauckler-Strickler's formula:

$$u = Kh^{2/3} S_{0x}^{1/2} \; ; \; v = Kh^{2/3} S_{0y}^{1/2} \tag{97, 98}$$

where K is the Strickler rougheness coefficient.
MacCormack's predictor-corrector scheme has an accuracy of second order in both space and time. The numerical integration of system is performed in the following form:

$$\mathbf{U}_{i,j}^p = \mathbf{U}_{i,j}^n - \frac{\Delta t}{\Omega_{i,j}} \sum_{r=1}^{4} [\mathbf{F},\mathbf{G}]_r^n \cdot \mathbf{n}_r \, \Delta L_r + \Delta t \mathbf{S}_{i,j}^n \tag{99}$$

$$\mathbf{U}_{i,j}^c = \mathbf{U}_{i,j}^n - \frac{\Delta t}{\Omega_{i,j}} \sum_{r=1}^{4} [\mathbf{F},\mathbf{G}]_r^p \cdot \mathbf{n}_r \, \Delta L_r + \Delta t \mathbf{S}_{i,j}^p \tag{100}$$

$$\mathbf{U}_{i,j}^{n+1} = \frac{1}{2}\left(\mathbf{U}_{i,j}^p + \mathbf{U}_{i,j}^c\right) \tag{101}$$

where p and c stand for predictor and corrector values. For each side (r = 1,…,4), $\mathbf{F}_r$ and $\mathbf{G}_r$ are obtained referring to upstream and downstream volumes alternately. The sequence is concluded in four time steps.

The MacCormack's scheme has been applied to the diffusive model (eqs. 79 -83) discretizing the mass conservation equation as in the equations (99 -101) while, for the momentum equations along the two directions x and y, the following expressions are applied both in the predictor and in the corrector steps:

$$\frac{1}{\Omega_{i,j}}\sum_{r=1}^{4}[F_2, G_2]_r^n \cdot \mathbf{n}_r \, \Delta L_r = gh_{i,j}^n (S_{0x} - S_{fx})_{i,j} \tag{102}$$

$$\frac{1}{\Omega_{i,j}}\sum_{r=1}^{4}[F_3, G_3]_r^n \cdot \mathbf{n}_r \, \Delta L_r = gh_{i,j}^n (S_{0y} - S_{fy})_{i,j} \tag{103}$$

where S_{fx} and S_{fy} are the friction slopes in x- and y- directions which can be calculated as:

$$S_{fx} = \frac{u\sqrt{u^2+v^2}}{K^2 \; h^{4/3}} \, ; S_{fy} = \frac{v\sqrt{u^2+v^2}}{K^2 \; h^{4/3}} \tag{104, 105}$$

and F_2, F_3, G_2, G_3 are the second and third components respectively of the vectors **F** e **G** according to equations (81-82).
Replacing the equations (104 - 105) in the equations (102 - 103) the values of the velocities, along the two directions of the motion u and v, may be easily calculated.
The kinematic model, in MacCormack's scheme the momentum equations are reduced to the equations (97 - 98) from which the values of the velocities are computed.
The codes of the schemes above described have been developed in Fortran 90 environment.

VALIDATION OF THE MODELS DEVELOPED BY THE LAMPIT LABORATORY (DEPARTMENT OF SOIL DEFENCE, UNIVERSITY OF CALABRIA) THROUGH NUMERICAL TESTS

After an accurate bibliographical review on both the overland flow models and the relative numerical applications, a number of significant test cases have been selected to validate the models developed in the second year by the laboratory LAMPIT (Department of Soil Defence - University of Calabria).
In particular the implemented schemes have been applied to simulate ideal overland flow processes for which an analytical solution or experimental data exist. In all the numerical applications the time step is determined by means of Courant-Friedrichs-Lewy criterion satisfying the numerical stability requirement.
In that context, it is necessary to underline that in presence of little values of water depth, as those that generally characterize the rainfall runoff process in the early stage of the phenomenon, the schemes can introduce some numerical anomalies with high values of the velocities (Esteves et al. 2000; Fiedler & Ramirez, 2000; Howes et at. 2006).
Such effect is more evident in the models based on the complete shallow water equations while it has been resulted negligible for the kinematic models. For that reason the value of the number of Courant used for these validation tests is quite small varying from 0.1 to 0.2.
Moreover, with reference to the schemes based on the complete equations of the shallow water, in some cases it has been necessary to introduce, in the schemes based on the complete shallow water equations, a very small water depth below which velocities have been calculated using a kinematic formulation.
The developed codes have been applied for simulating the Test 1, whose geometric and hydraulic characteristics have been brought in the previous paragraph. The numerical results have been

compared with an analytical solution, proposed in the literature, based on the kinematic hypothesis.
In all the simulations the computational domain has been divided in square cells of side equal to 5 m and the number of Courant has been set equal to 0.2.
As shown in the Figure 26 the results obtained by all schemes are in good agreement with the analytical solution.
In particular MacCormack's scheme, applied to the kinematic model, results the most accurate one because it is in a good agreement with the analytical solution obtained using the kinematic hypothesis while HLL scheme, applied to both the complete model and the kinematic model, presents a light increase of the outflow discharge.

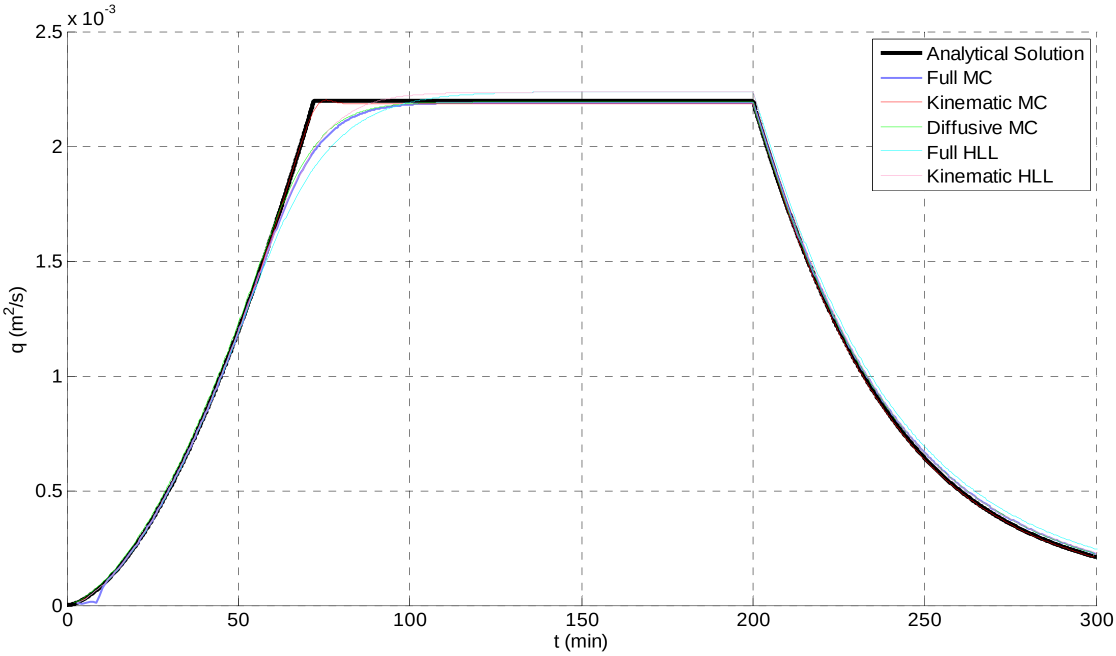

Figure 26 – Unit discharge outflow hydrographs for test 1: comparison between the analytical solution and numerical simulations

The implemented schemes have been applied also for simulating Test 4a.
In all the simulations the computational domain has been divided in square cells of side equal to 0.1 m and the number of Courant has been set equal to 0.2.
In the figure (27) the runoff obtained by all schemes is compared with the analytical solution. Once again, the arrival time and the maximum discharge are in good agreement with the analytical solution.
For that test, the numerical solutions of the schemes based on the complete shallow water equations have not been reported due to the presence of some numerical anomalies. In order to avoid the generation of these anomalies the simulations would require very small values of the Courant number and the computational time results thus very burdensome.

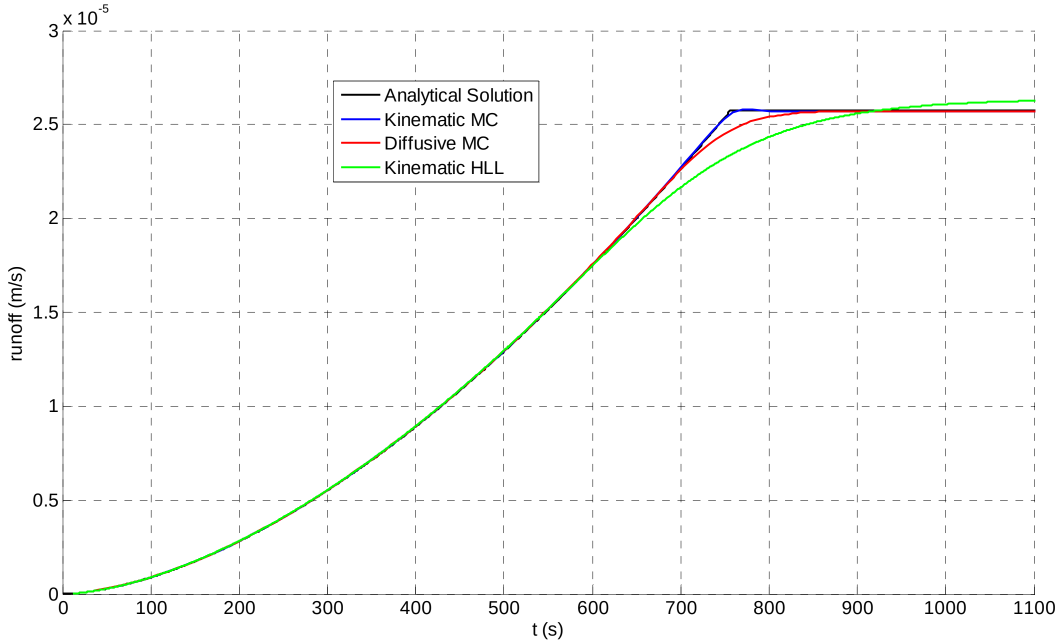

Figure 27 – Runoff hydrographs relative to test 4a: comparison between analytical solution and numerical simulations

Tests 6a and 6b have been used as representative of those situations in which a time variable rainfall intensity occurs over a tilted plane. The numerical results for the aforementioned tests are shown in the figures 28 and 29. In these tests the numerical runoff obtained by simplified models are compared with the solutions obtained by the complete models.

The computational domain for both the tests has been divided in the cells of dimensions 0.1 x 0.1 m while the Courant number is set to 0.1. The figures show a good accuracy of the diffusive model in comparison to the complete model.

It is interesting to observe that in the Test 6a the results based on the kinematic approximation of the equations show that, in this case, the numerical depth gradient contribution is not negligible in comparison to the bottom slope. For that reason, the differences between the numerical solutions of the kinematic models and those of complete models are due to neglecting such terms (figure 28). On the contrary, the solutions of the implemented schemes are very similar when applied to the Test 6b (figure 29).

For both the cases the numerical results are in agreement with those presented by other authors.

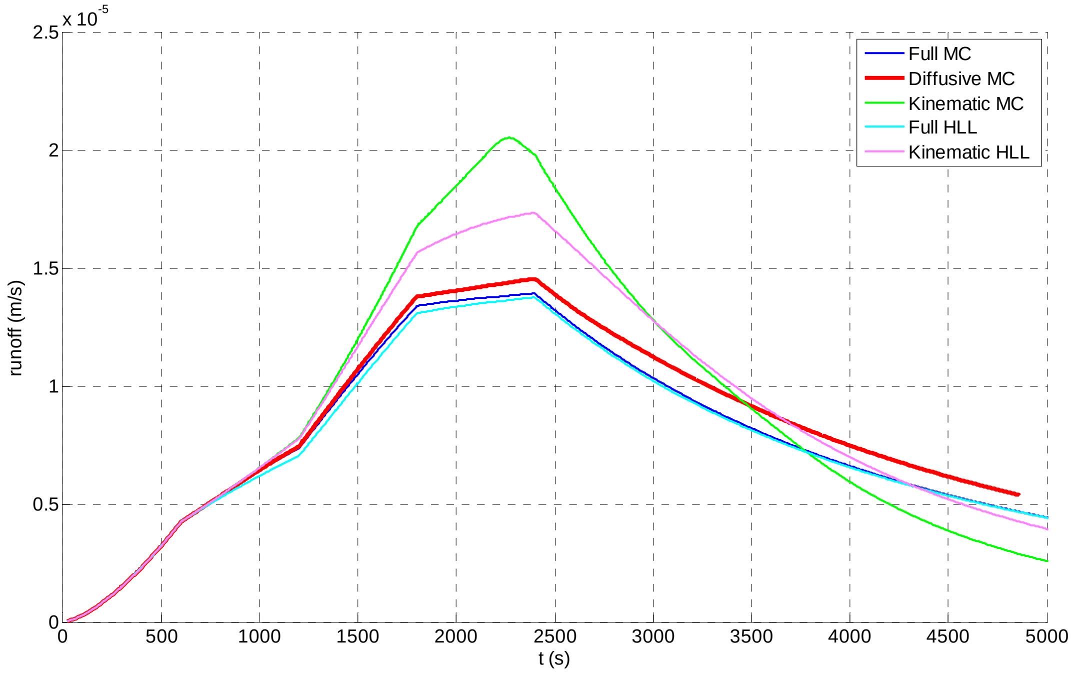

Figure 28 – Runoff hydrographs for test 6a: comparison among the solution obtained by the numerical simulations.

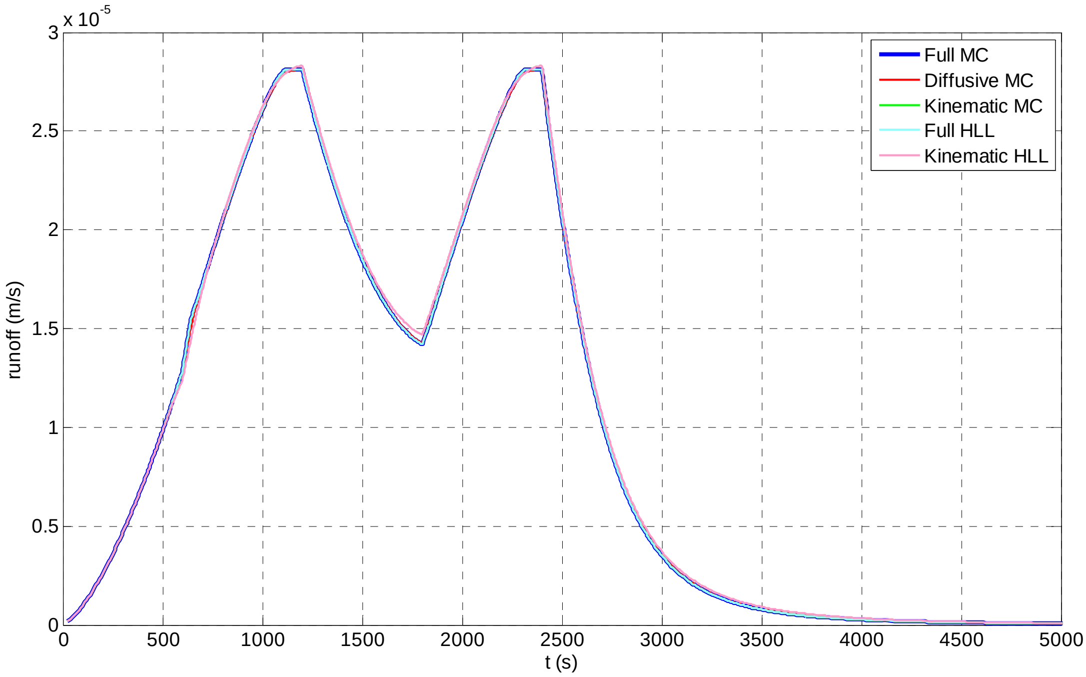

Figure 29 – Runoff hydrographs for test 6b: comparison among the solution obtained by the numerical simulations.

A number of interesting tests concern the simulation of the overland flow process caused by a space varying rain on three-plane cascade (Test 7a, 7b, 7c).
In these tests the numerical solutions of the schemes have been compared with the experimental data in terms of both the channel outlet discharge and water depth profiles in different instants of time.

The computational domain for both the tests has been divided in the cells of dimensions 0.1 x 0.1 m. In the Figures 30-34 the comparison between the numerical results and the experimental data is shown; in particular, for each test, the water depth profiles refer to the time instant in which the rain ends (30 s, 20 s, 10s).
The discharge hydrographs obtained by using the different schemes are in good agreement with the experimental data (Figures 30, 32 34); in fact, the differences between the numerical solutions and the experimental data in both the maximum discharge and the arrival times seem to be quite small. On the contrary, the simulated water depth profiles, especially those referred to the simplified model, show significant differences with the observed ones.
It has to be said that the main goal in the simulation of the overland processes over an hillside is to evaluate the runoff as discharge hydrographs that comes down from hillsides themselves because they represent, from a modellistic point of view, a lateral inflow contribution in terms of volume for unit of time; consequently there is the need of an accurate computation in terms of the flow discharges rather than in the water depths.
Once again the numerical results achieved by the implemented codes are similar to those presented in literature by other authors for the same tests. The most accurate solutions are those computed discretizing the complete models using both the HLL scheme and the MacCormack scheme. For the latter scheme (Figures 33 -35), some numerical oscillations appear due to the presence of a discontinuity of the water depth profile. Such aspect is characteristic of the second order accurate MacCormack scheme, and it will be correct introducing an artificial viscosity term, as already stated in previous LAMPIT (Department of Soil Defense - University of Calabria) papers.
The models provide the better performance when used to the simulation of test in which the longer rain duration occur (Test 7a –rain duration 30 s); that happens probably because, as stated above, the presence of very small values of water depths introduce some numerical inaccuracies during the early stage of the phenomenon.

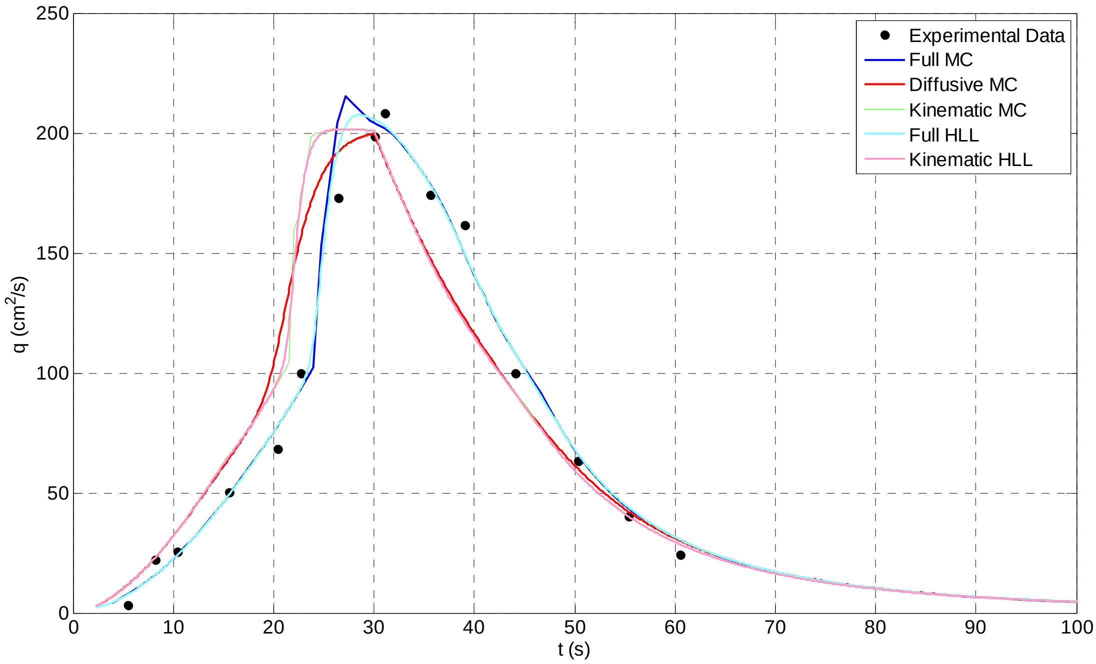

Figure 30 – Flood wave at the channel outlet: comparison between experimental points and numerical simulations. (Rain duration *t*=30s)

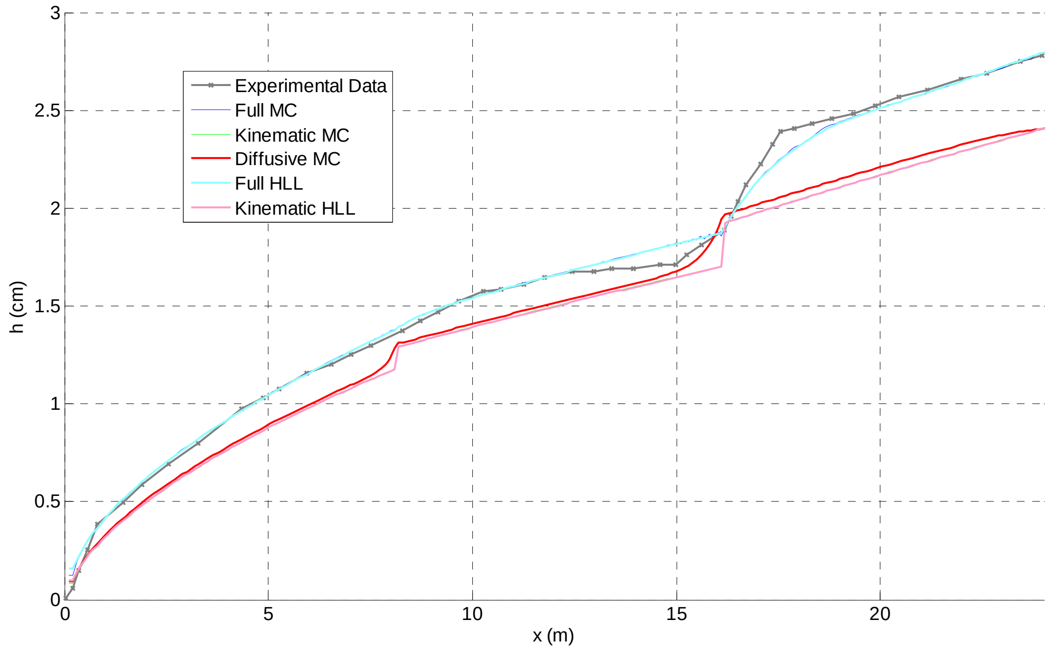

Figure 31 – Longitudinal water depths profile at the end of the rainfall input (t=30s): Comparison between experimental data and numerical results

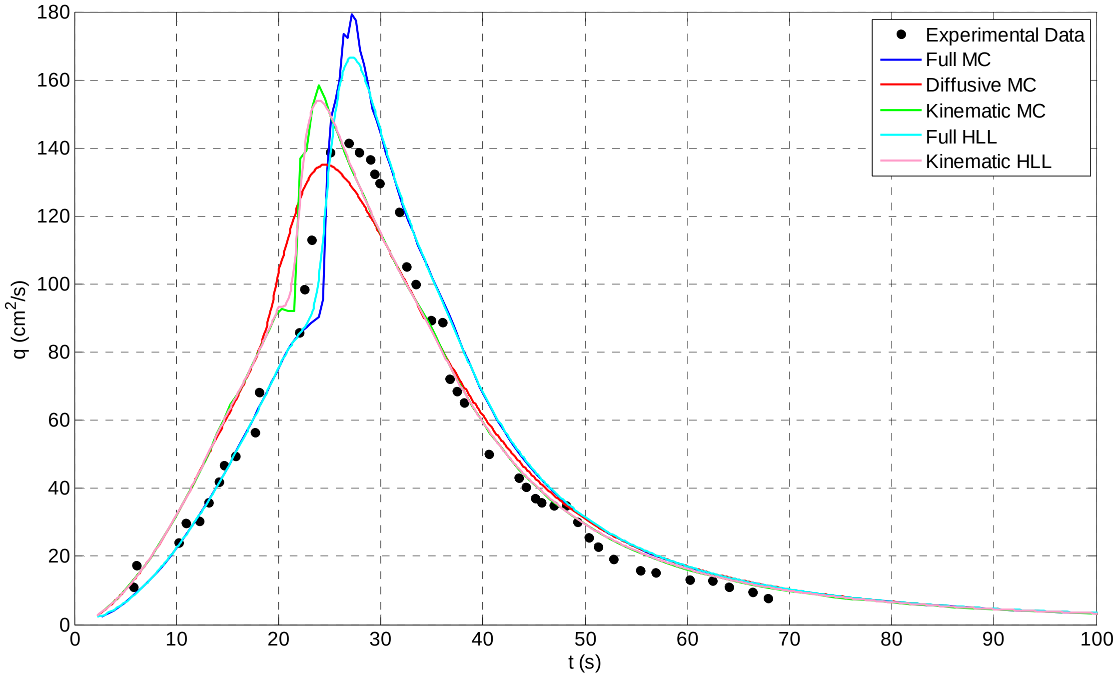

Figure 32 – Flood wave at the channel outlet: comparison between experimental points and numerical simulations. (Rain duration t=20s)

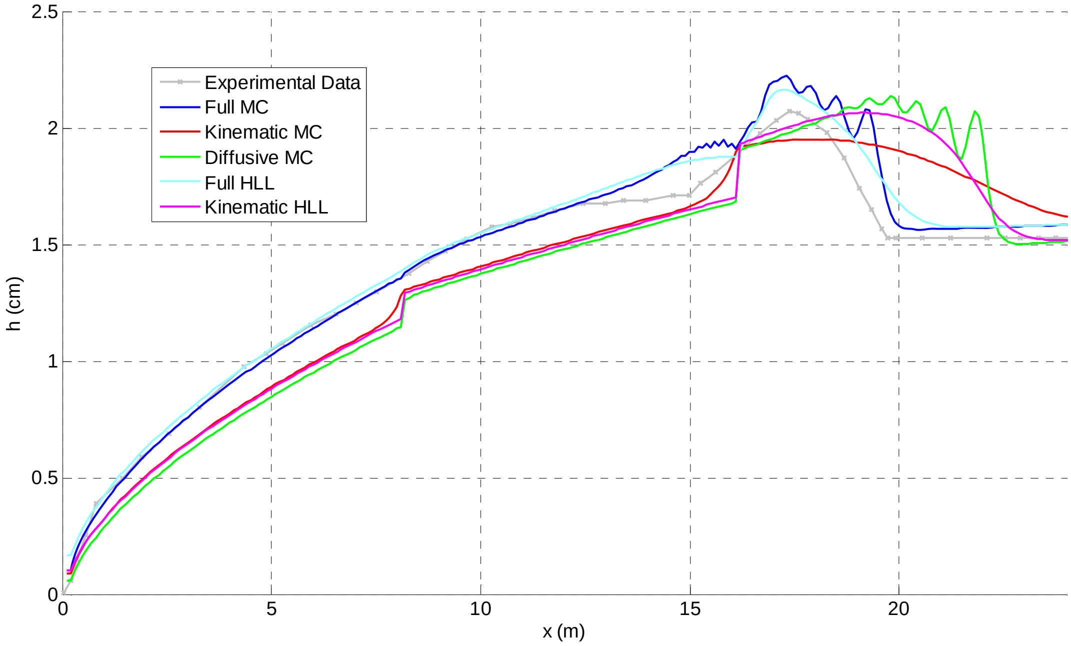

Figure 33 – Longitudinal water depths profile at the end of the rainfall input (t=20s): Comparison between experimental data and numerical results

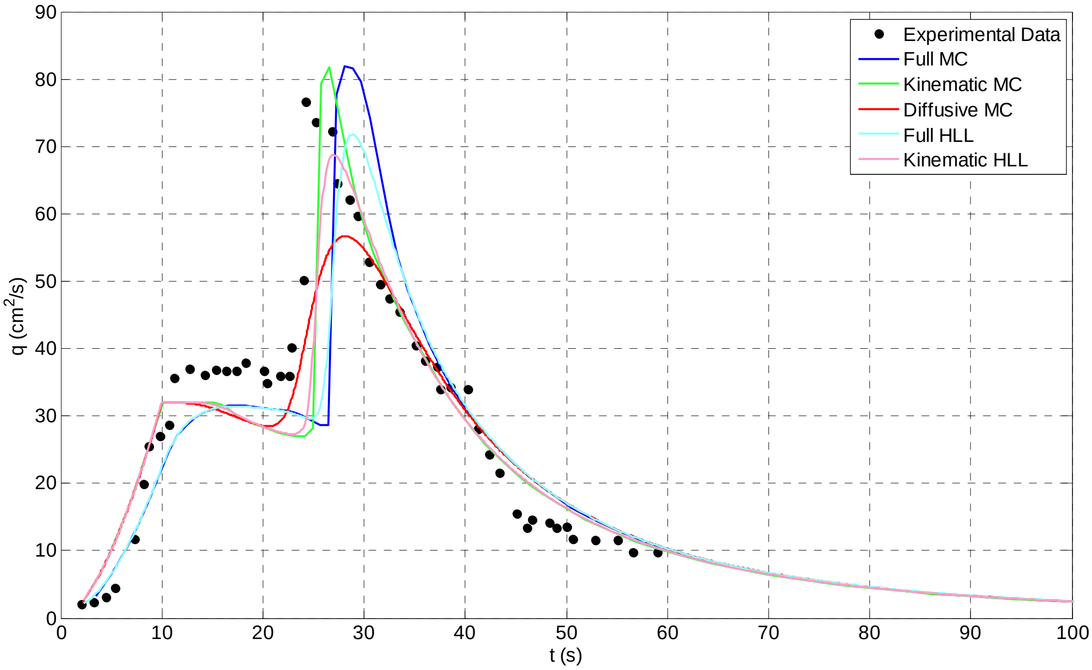

Figure 34 – Flood wave at the channel outlet: comparison between experimental points and numerical simulations. (Rain duration *t*=10s)

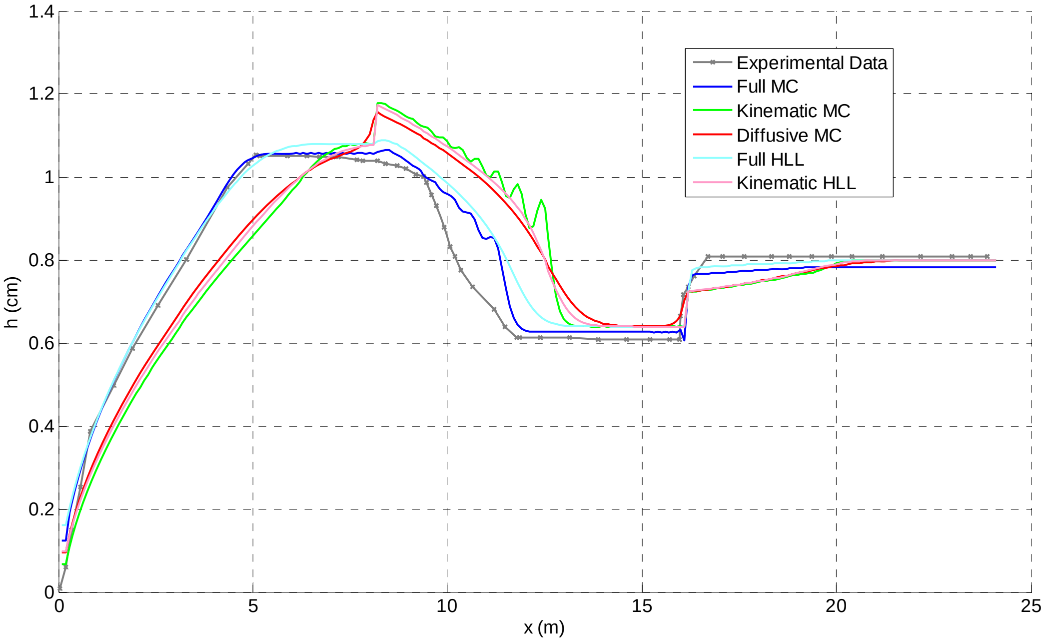

Figure 35 – Longitudinal water depths profile at the end of the rainfall input (t=10s): Comparison between experimental data and numerical results

The study of overland flow processes in real situation often refers to a large (wide) area; as a consequence, in order to avoid a significant increase in terms of both computational times and memory storage, the computational domain is obtained by using very coarse cells.

In this context, an analysis on the accuracy of the numerical solutions in relationship to the size of computational cell has been performed. Therefore the effects of meshing size on numerical results were also analyzed.

In Figures (36-40) the comparisons of the discharge hydrographs obtained by the implemented schemes by using different cell sizes (dx = 0.1 m, dx = 1 m, dx = 2 m, dx = 4 m) are shown.

It is interesting to note that an increase of the cell size in the MacCormack scheme, used to solve the complete unsteady flow and the simplified flow models, does not significantly change the accuracy of the solution (Figs 36-40). On the contrary, the results obtained using the HLL scheme are quite sensitive to the cell size and, in particular, they are less accurate when the cell size increase. This is mainly due to the accuracy of the MacCormack scheme while HLL scheme is first order accurate.

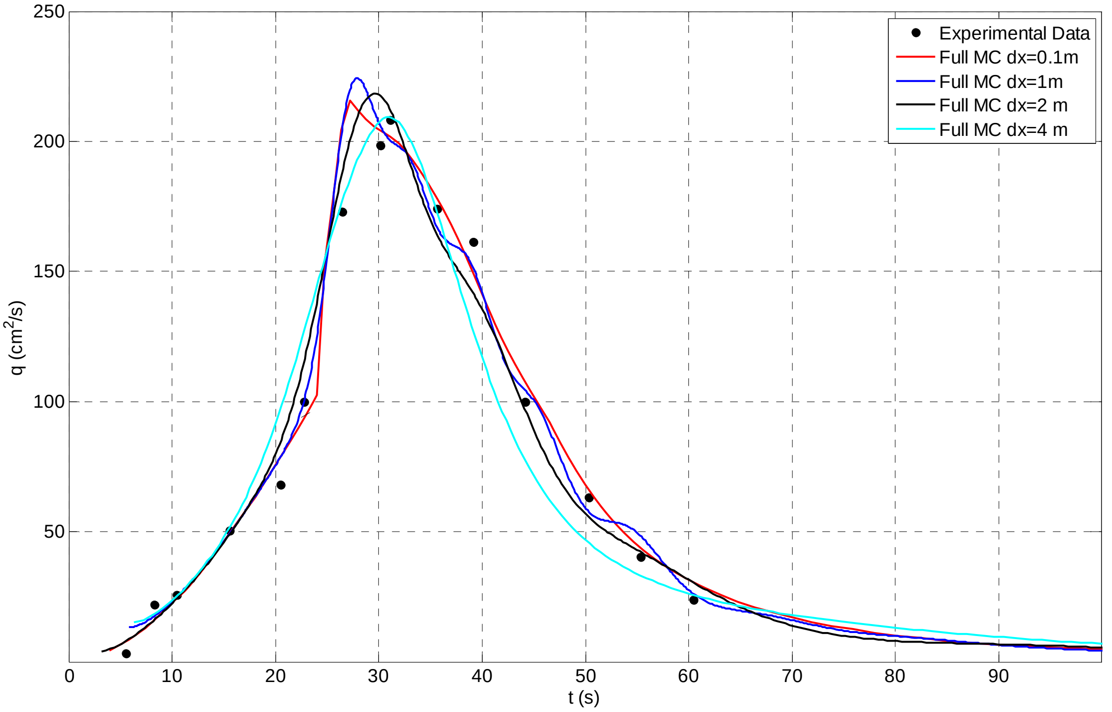

Figure 36 – Flood wave at the channel outlet: influence of mesh size on the computed hydrographs using MacCormack scheme applied to the full unsteady flow model (Rain duration *t*=30s)

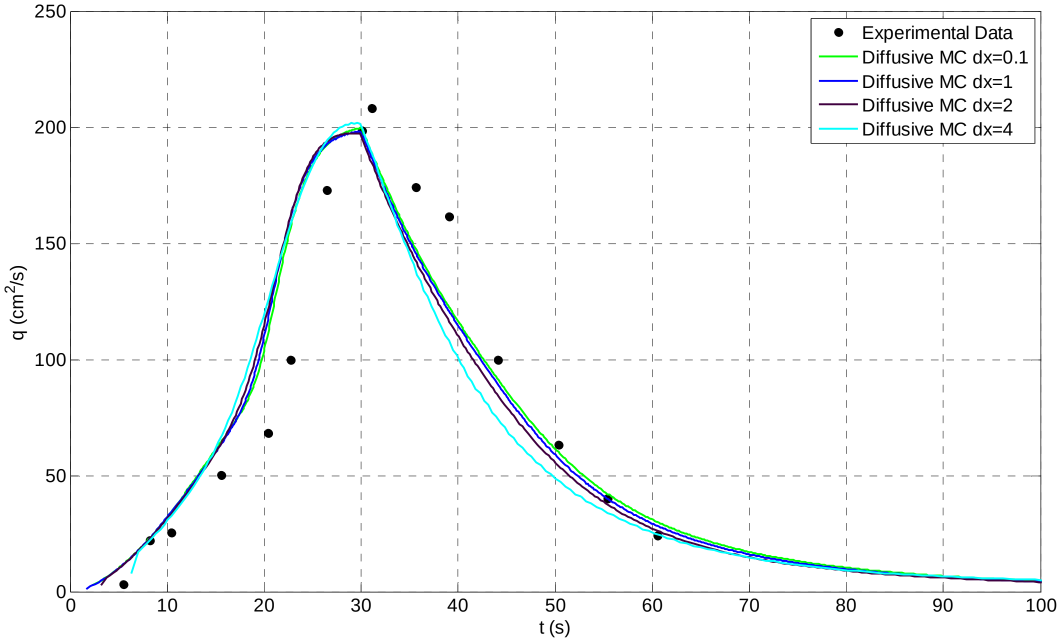

Figure 37 – Flood wave at the channel outlet: influence of mesh size on the computed hydrographs using MacCormack scheme applied to the diffusive flow model (Rain duration *t*=30s)

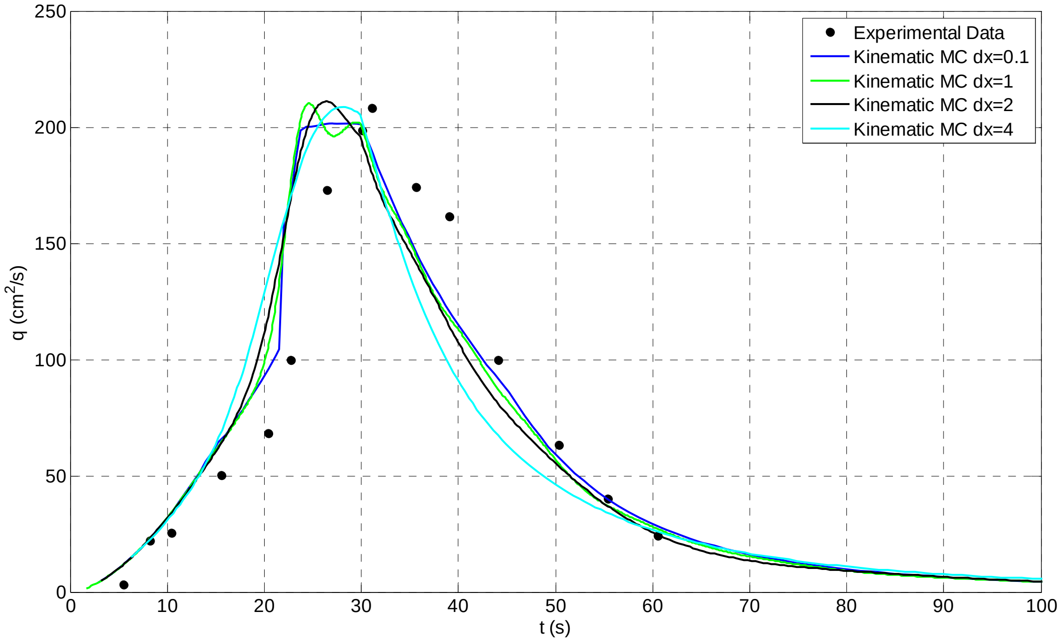

Figure 38 – Flood wave at the channel outlet: influence of mesh size on the computed hydrographs using MacCormack scheme applied to the kinematic model (Rain duration *t*=30s)

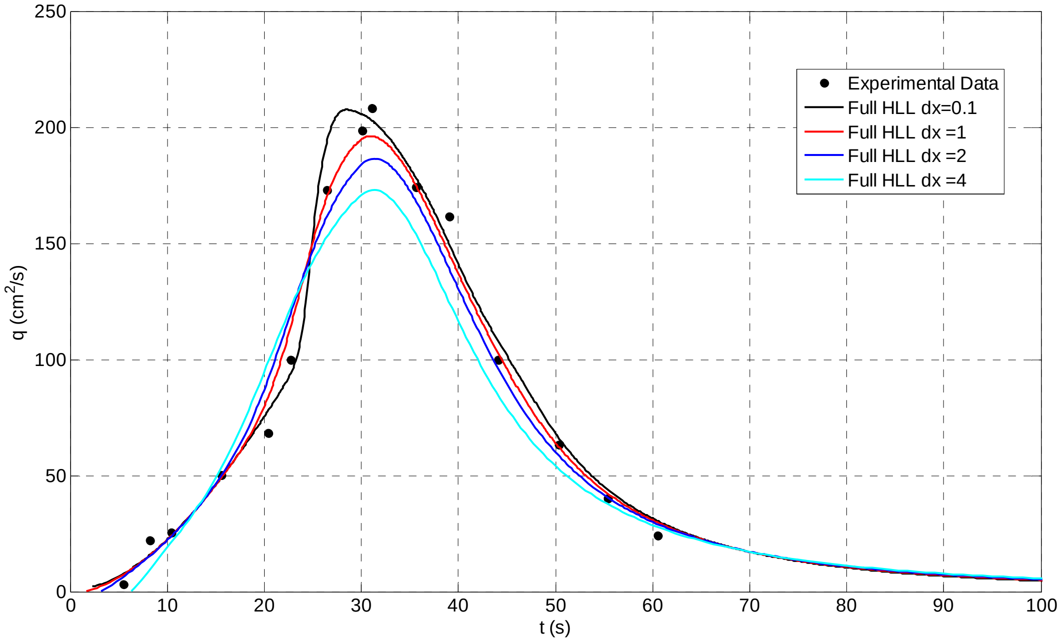

Figura 39 – Flood wave at the channel outlet: influence of mesh size on the computed hydrographs using HLL scheme applied to the full unsteady flow model (Rain duration *t*=30s)

The last test used to validate the models developed by LAMPIT laboratory is an overland flow problem in which a heavy rain falls on two hypothetical hillside at whose bottom a constant slope channel is located.(Test 8). The Figures 41 - 42 show the results obtained by the different schemes compared with the analytical solution in terms of both the outflow discharge coming down for each hillside (Figure 41) and the discharge at the channel outlet (Figure 42). In both

figures the numerical results obtained by the implemented schemes are in good agreement with the analytical solution.
Particularly the achieved numerical solutions are in a good agreement each other except for a little diffusion caused by the first order HLL scheme with a low increase of the outflow discharge.

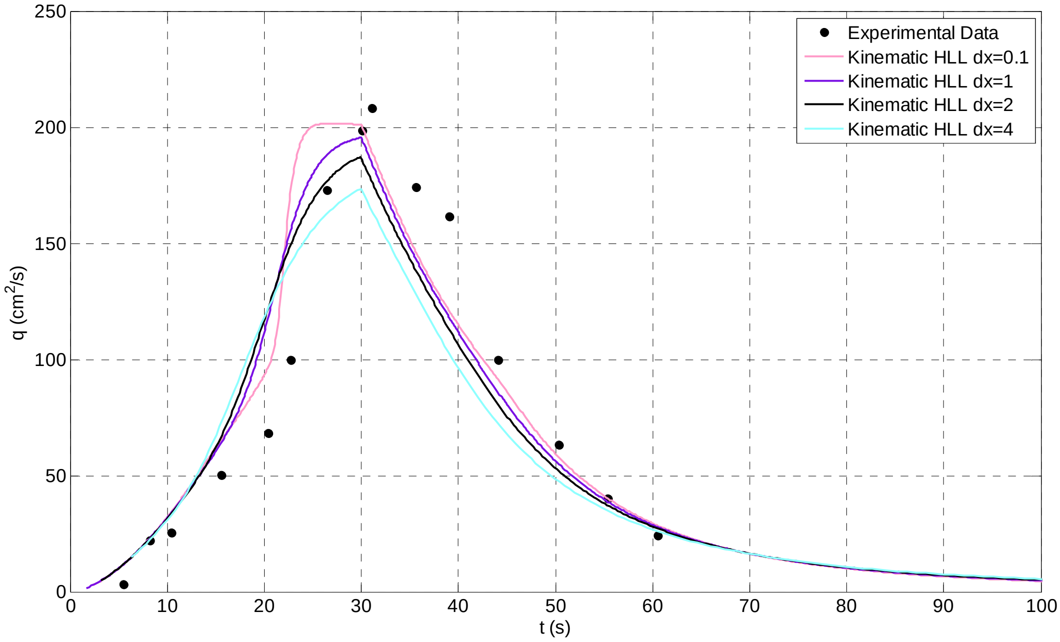

Figure 40 – Flood wave at the channel outlet: influence of mesh size on the computed hydrographs using HLLscheme applied to the kinematic flow model (Rain duration *t*=30s)

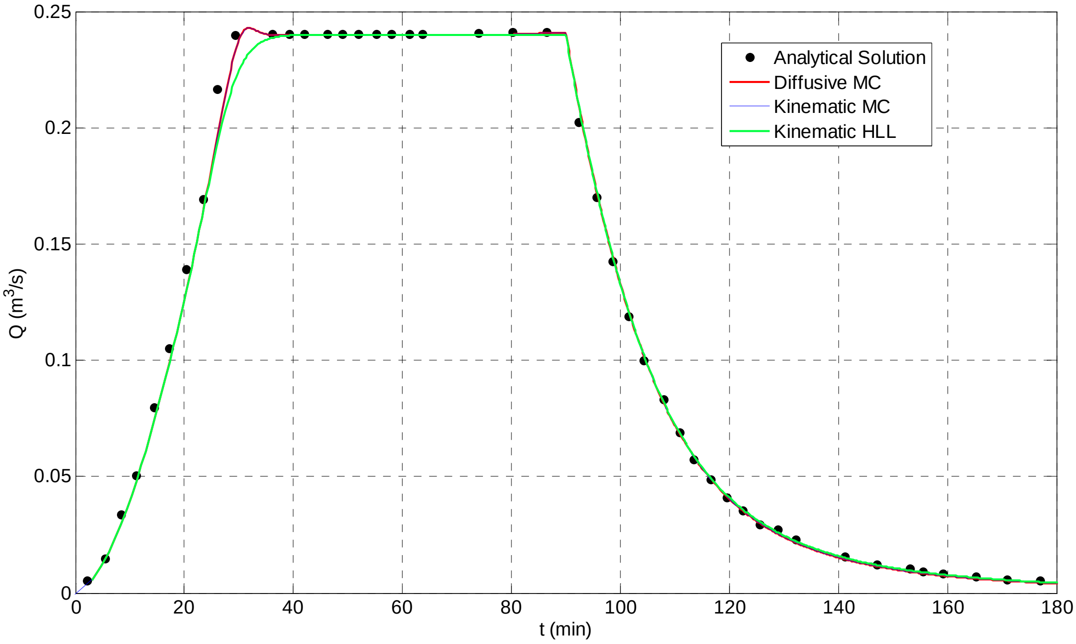

Figure 41 – Flood wave at the bottom of the hillside (test 8): comparison between numerical results and analytical solution

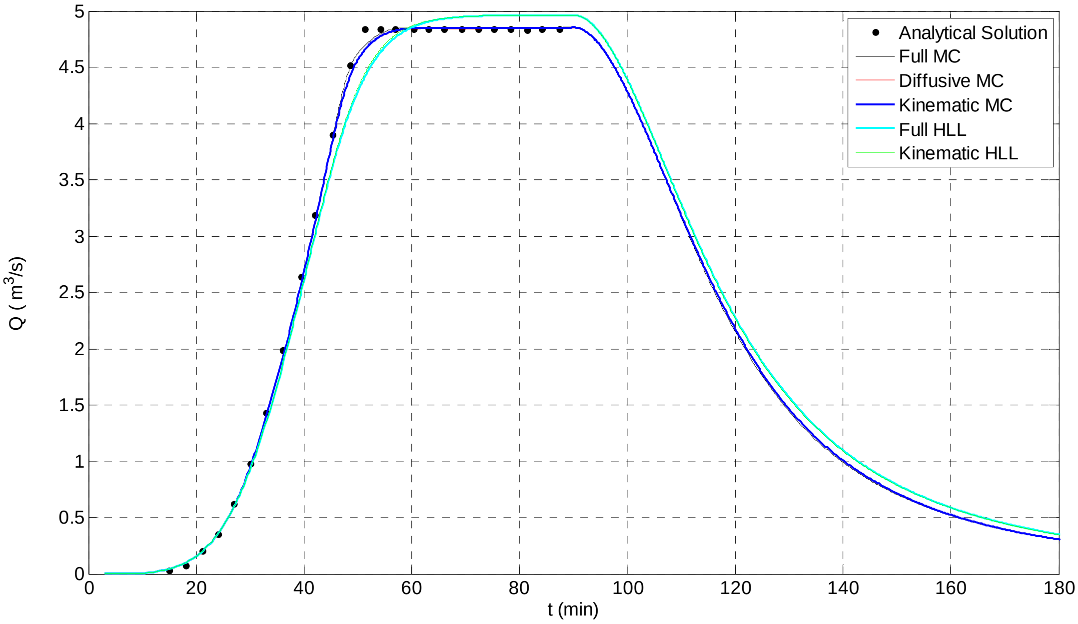

Figure 42 – Flood wave at the channel outlet (test 8): comparison between numerical results and analytical solution

CONCLUSIONS

The main difficulty that one encounters during the overland flow simulation deals with the propagation of very low water depths which cause some physical-numerical anomalies as high values of the velocities; that problem arises using the shallow water equations written both in their complete and simplified (diffusive wave, kinematic wave) form.
Such effect is more evident when the complete formulation of the equations is used while it is quite negligible in the kinematic models in which the velocities are calculated using the Gauckler-Strickler's formula.
It is important to notice that in the simulation of overland flow events the convective inertial terms in the momentum equations are significant lower than the values of the topographic surface slope in those situation in which a strong altimetrical gradient occurs. Whenever that eventuality happens, it is justified to neglect these terms in order to avoid an useless increase of the computational times
Therefore a comparative analysis on the accuracy of the results obtained by the simplified models (diffusive model and kinematic model) and those obtained by complete model has been carried out.
From the numerical results it is possible to note that both the simplified models give very similar results to those obtained by the complete model and in good agreement with experimental data or with the analytical solutions.
Particularly the numerical solutions show that the models perform better in reproducing the values of discharges rather than water depths. On the other hand the main goal in the simulation of the overland processes over an hillside is to evaluate the runoff that comes down from hillsides themselves as discharge hydrographs because they represent, from a modellistic point of view, a lateral inflow contribution in terms of volume for unit of time; consequently there is the need of an accurate computation in terms of the flow discharges rather than the water depths.

Among the implemented schemes, the second order accurate MacCormack's scheme gives best results and it is the more simple scheme to discretize both the complete model and the simplified (diffusive and kinematic) models. Nevertheless such scheme introduces some numerical anomalies in presence of discontinuity on the water depth profile. Subsequently the use of artificial viscosity terms will avoid the generation of these numerical anomalies.
The study of overland flow processes in real situation often refers to a large (wide) area; as a consequence, in order to avoid a significant increase in terms of both computational times and memory storage, the computational domain is obtained by using very coarse cells.
In this context, an analysis on the accuracy of the numerical solutions in relationship to the size of computational cell has been performed. Therefore the effects of meshing size on numerical results were also analyzed. This study has showed that MacCormack's scheme maintains a good accuracy in the numerical results with the cell size increase while HLL scheme gives less accurate results to the increase of the cell size.
These aspects have subsequently been studied in detail applying the implemented schemes to simulate a two dimensional test with the purpose to implement a robust code easily adaptable to any type of topography.
This study has shown that, for all the simulated tests in which the conditions of applicability of the simplified models are satisfied, the numerical schemes implemented allow satisfactory performances when using simplified systems (79) and (84).

REFERENCES

Ames, W. R. (1992). *Numerical methods for partial differential equations*. Academic Press, New York.

Balloffet A., Scheffler M.L. (1982). "Numerical Analysis of the Teton Dam Failure Flood", *J. Hydr. Res*., 20 (4): 317-328.

Bates, P.D., De Roo, A.P.J. (2000). "A simple raster-based model for flood inundation simulation". *Journal of Hydrology* 236 (1–2), 54–77.

Bladé, E., Gómez, M., Dolz, J. (1994). "Quasi-two dimensional modelling of flood routing in rivers and flood plains by means of storage cells", *Modelling of Flood Propagation Over Initially Dry Areas*, a cura di Molinaro P. e Natale L., 156-170.

Braschi, G., Gallati, M. (1989) "Simulation of a levee-breaking submersion of planes and urban areas", HYDROCOMP'89, *Proc. Int. Conf. on Computational Modelling and Experimental Methods in Hydraulics*, Elsevier Applied Science, 117-126.

Costanzo, C., Macchione F. (2004). "Prestazioni di alcuni schemi bidimensionali ai volumi finiti per problemi di dam break". Atti del *XXIX Convegno di Idraulica e Costruzioni Idrauliche*, Trento, 2004, Ed. Bios:Cosenza, 2004, 1: 829-836.

Costanzo, C., Macchione, F. (2005a). "Comparison of two-dimensional finite volume schemes for dam break problem on an irregular geometry". *Proceedings of the XXXI IAHR Congress*, Seoul, Korea 2005, Theme D, 3372-3381. CD-ROM.

Costanzo, C., Macchione, F. (2005b). "Simulazioni numeriche bidimensionali di fenomeni di dam break". Atti del I *Workshop Modeci*, Arcavacata di Rende, 30 - 31 marzo, 2004, AGM s.r.l.:Castrovillari, 2005, pp. 179-190.

Costanzo C., Macchione F. (2006). "Tecniche numeriche per la simulazione della propagazione di piene su fondo irregolare". In *Tecniche per la Difesa dall'Inquinamento*, G. Frega (a cura di), Editoriale Bios, Cosenza, 2006, Vol. 26, pp. 423-440.

Davis, S. F. (1988). "Simplified second-order Godunov-type method". *Journal of Scientific and Statiscal Computing, SIAM*, 9: 445-473.

Defina, A., D'Alpaos, L., Matticchio, B.(1994). "A new set of equations for very shallow water and partially dry area suitable to 2D numerical models", *Proceedings of Modelling of Flood Propagation Over Initially Dry Areas,* a cura di Molinaro, P. e Natale, L., ASCE, 72-81.

Di Giammarco, P., Todini, E., Lamberti, P. (1996). "A conservative finite elements approach to overland flow: the control volume finite element formulation". *Journal of Hydrology*, Elsevier, 175: 276.291.

Esteves, M., Faucher, X., Galle, S., Vauclin, M. (2000). "Overland flow and infiltration modelling for small plots unsteady rain: numerical results versus observed values". *Journal of Hydrology*, Elsevier, 228: 265-282.

Feng, K., Molz, G. J. (1997). "A 2-D, diffusion-based, wetland flow model". *Journal of Hydrology*, Elsevier, 196: 230-250.

Fiedler, F. R., Ramirez, J. A. (2000). "A numerical method for simulating discontinuos shallow flow over an infiltrating surface". *International journal of numerical method in fluids*, 32: 219-240, John Wiley & Sons Ltd.

Frega, G., Macchione, F., Rinaldi, L. (1999). "Schema quasi–bidimensionale per la simulazione degli allagamenti di aree urbane", L'idraulica nei problemi di protezione del territorio, a cura di G. Frega, Quaderni di Idrotecnica (12), Editoriale Bios, , 61-77.

Gandolfi, A., Savi, F. (2000). "A mathematical model for the coupled simulation of surface runoff and infiltration". *Journal of Agricultural Engineering Research*, 75: 49-55, Silsoe Research Institute.

Gottardi, G., Venutelli, M. A. (1993). "A control-volume finite-element model for two-dimensional overland flow". *Advances in Water Resources*, Elsevier, 16: 277-284.

Gottardi, G., Venutelli, M. (2008). "An accurate time integration method for simplified overland flow models". *Advances in Water Resources*, Elsevier, 31: 173-180.

Govindaraju, R. S., Jones, S. E., Kavas, M. L. (1988). "On the diffusion wave model for overland flow. 1. Solution for Steep Slopes". *Water Resources Research*, 24 (5): 734-744.

Horritt, M.S., Bates, P.D. (2001). "Effects of spatial resolution on a raster based model of flood flow". *Journal of Hydrology* 253 (1–4), 239–249.

Hunter, N.M., Horritt, M.S., Bates, P.D., Wilson, M.D., Werner, M.G. F., (2005). "An adaptive time step solution for raster-based storage cell modelling of floodplain inundation". *Advances in Water Resources* 28 (9): 975–991.

Howes, D.A., Abrahams, A. D., Pitman, E. B. (2006). "One- and two-dimensional modelling of overland flow in semiarid shrubland, Jornada basin, New Mexico". *Hydrological processes*, Wiley, 20: 1027.1046.

Hromadka, II T.V., Lai C. (1985). "Solving the two-dimensional diffusion flow model". *Proceedings of Special Confrence Sponsored by Hydraulic Division*, ASCE, Lake Buena Vista Fla, 555-561.

Hromadka II, T.V., Yen, C.C.(1986). "A Diffusion Hydrodinamic Model (DHM)", *Adv. Water Resources*, 9: 118-170.

Iwagaki, Y. (1955). "Fundamental studies on the runoff analysis of characteristics". *Bull. 10, Disaster Prevention Research Institution*, Kyoto University, Kyoto, Japan.

Jaber, F. H., Mothar, R. H. (2003). "Stability and accuracy of two dimensional kinematic wave overland flow modeling". *Advances in Water Resources*, Elsevier, 26: 1189-1198.

Kazezyilmaz-Alhan, C., Medina, M. A. (2007). "Kinematic and diffusion waves: analytical and numerical solutions to overland and channel flow". *Journal of Hydraulic Engineering*, ASCE, 133(2): 217-228.

Laura, A.M., Wang, J.D.(1984) "Two-dimensional flood routing on steep slopes", *J. Hydr. Div*., ASCE, 110 (8): 1121-1135.

Liang, D., Falconer, R. A., Lin, B. (2007). "Coupling surface and subsurface flows in a depth averaged flood wave model".*.Journal of Hydrology*, Elsevier, 337: 147-158.

Macchione, F. (1994). "Application of the Muskingum-Cunge method for dam break flood routing". In *Hydraulic Engineering Software,* 2(V): 133-140.

Macchione F. , Viggiani G. (2000). "Applicabilità dei modelli semplificati di propagazione delle piene generate da rottura di dighe nei corsi d'acqua calabresi", *Memorie e Studi del Dipartimento di Difesa del Suolo*, Università della Calabria, Editoriale Bios, Cosenza, Vol. 331.

Macchione, F. (2008). " Relazione Generale sul Tema 3. Allagamenti di aree urbane". In *Tecniche per la Difesa dall'Inquinamento*, G. Frega (a cura di), Editoriale Bios, Cosenza, 2008, Vol. 29, pp. 161-184.

Maione, U., Mignosa, P., Tanda, M.G. (1986). "Influenza di un rilevato autostradale su un'inondazione", *Ingegneria Ambientale*, 15 (1), gennaio.

Molinaro, P., Di Filippo, A., Ferrari, F. (1994). "Modelling of flood wave propagation over flat complex topography in presence of different infrastructures", *Proceedings of Modelling of Flood Propagation Over Initially Dry Areas,* a cura di Molinaro, P. e Natale L., ASCE, 209-228.

Moramarco, T., Singh, V. P. (2002). "Accuracy of kinematic wave and diffusion wave for spatial-varying rainfall excess over a plane". Hydrological Processes, Wiley, 16: 3419:3435.

Morris, E. M., Woolhiser, D. A. (1980). "Unsteady one-dimensional flow over a plane: Partial equilibrium and recession hydrograph". *Water Resources Research*, 16 (2): 355-360.

Natale, L., Savi, F.(1991). "Espansione di onde di sommersione su terreno inizialmente asciutto", *Idrotecnica*, 6: 397-406.

Natale, L.(1988) "*Mappatura del rischio di inondazione*". Rapporto G.N.D.C.I.

Reitano, B. (1992). "Modello bidimensionale per la simulazione di inondazioni fluviali", Atti del *XXIII Convegno di Idraulica e Costruzioni Idrauliche*, 3: 321-336.

Singh, V. P. (1996). *Kinematic wave modelling in water resources: Surface water hydrology.* John Wiley & Sons: New York.

Stephenson, D., Meadows, M.E. (1986). *Kinematics Hydrology and Modelling*. Elsevier, Amsterdam.

Tsai, T.L., Yang, J.C. (2005). Kinematic wave modeling of overland flow using characteristics method with cubic-spline interpolation. *Advances in Water Resources*, Elsevier, 28:661-670.

Tucciarelli, T., Aronica, G., La Loggia, G. (1995). "Studio della propagazione delle onde di piena su alveo inizialmente asciutto". *Idrotecnica*, 5, 299-307.

Turner, A. K, Chanmeesri, N. (1984) "Shallow flow of water through non-submerged vegetation". *Agricultural Water Management*, 8: 375-385.

Woolhiser, D. A., Liggett, J. A. (1967). "Unsteady, one-dimensional flow over a plane - The rising hydrograph". *Water Resources Research*, 3 (3): 753-771.

Woolhiser, D. A. (1974). "Unsteady free-surface flow problem". *Proceedings of Institute on Unsteady Flow in Open Channels*, Colorado State University, Fort Collins, 195-213.

Yu, D., Lane, S.N. (2006a). "Urban fluvial flood modelling using a two-dimensional diffusion-wave treatment, part 1: mesh resolution effects", *Hydrological Processes*, Wiley, 20:1541-1565.

Yu, D., Lane, S.N. (2006b). "Urban fluvial flood modelling using a two-dimensional diffusion-wave treatment, part 2: development of a sub-grid-scale treatment", *Hydrological Processes*, Wiley, 20:1567-1583.

Yu, D., Lane, S.N. (2007). "Coupling of a 2D diffusion based flood inundation model to a 1D river flow for urban flood inundation modelling". *Proceedings of 32nd Congress of IAHR*, Venice (Italy).

Zanobetti, D., Lorgeré H. (1968). "Le modèle mathématique du delta du Mékong*", La Houille Blanche*, n. 1-4-5, 17-30, 255-269, 363-378.

Zanobetti, D., Lorgeré, H., Preissmann, A., Cunge J.A. (1970). "Mekong delta mathematical model program construction". *Journal of the Waterways and Harbors Division*, ASCE, 96 (WW2), May, 181-199.

.