CENTRO EURO-MEDITERRANEO
PER I CAMBIAMENTI CLIMATICI

CENTRO EURO-MEDITERRANEO
PER I CAMBIAMENTI CLIMATICI

Technical Reports

ISC – Divisione Impatti sul Suolo e sulle Coste

Final document on the second year third activity: "Analysis of the hydrometeorological chain performances on a real basin"

Prof. Francesco Macchione*

Ing. Pierfranco Costabile*

Ing. Carmelina Costanzo*

* *LAMPIT (LAboratorio di Modellistica numerica per la Protezione Idraulica del Territorio) – Dipartimento di Difesa del Suolo – Università della Calabria*

Centro Euro-Mediterraneo
per i Cambiamenti Climatici
www.cmcc.it

March 2009 ■ TR

Final document on the second year third activity: "Analysis of the hydrometeorological chain performances on a real basin".

Abstract

This report is the final document related to the second year, third activity whose title is: "Analysis of the hydrometeorological chain performances on a real basin".

The purpose of the collaboration between LAMPIT (Department of Soil Defence, University of Calabria) and CMCC is to develop an hydrometeorological chain in order to obtain a reliable tool in the context of flood evolution prediction able to provide quantitative information of practical importance within the civil protection activities.

The LAMPIT contribution to the project concerns the mathematical description of both the generation and propagation of flood events at basin scale. The work here presented has been carried out in close cooperation with CIRA researchers (dr. Pasquale Schiano and dr. Paola Mercogliano). The evaluation in time and magnitude of the overland flow phenomena caused by rainfall is very important in a variety of environmental and hydraulics situations, such as flood predictions, surface erosion, waste water treatment, irrigation and drainage. The mathematical representation of the flow processes is based on the fully dynamic shallow water equations. The solution of these equations, excluding some simplified cases, can be obtained by numerical integration only. Many schemes based on fully dynamic and simplified shallow water equations have been proposed in literature. Some of these schemes are implemented in the LAMPIT laboratory and applied to simulate simple cases of overland flow as already presented in the previous reports. In this period the implemented code has been applied to simulate the surface runoff over a real topography. In literature few articles deal with the simulation of real events and many difficulties are reported in each of them. Indeed the presence of small water depth over high slope and irregular topography may induce numerical anomalies. In this work some numerical techniques have been implemented to prevent these problems. Finally the implemented scheme has been applied to simulate the surface runoff over three different real watersheds.

The work has been organized as follows:

- Analysis of the models proposed in the technical literature with particular reference to the simulation of real event.
- Development and implementation of numerical scheme and techniques
- Simulation of overland flow events on real watersheds

Keywords: Hydrometeorological chain, Flood propagation

JEL Classification:

Address for correspondence:
Prof. Francesco Macchione
Laboratorio LAMPIT
Dipartimento di Difesa del Suolo – Università della Calabria
Ponte P.Bucci – Cubo 42/B
87036, Rende (CS) .
E-mail: f.macchione@unical.it

Analysis of the models proposed in the technical literature with particular reference to the simulation of real event

The dynamics of rainfall, infiltration, surface runoff and erosion processes would be more easily understood with a correct, adequate and scale-variant mathematical representation of these interactive processes. Surface runoff (overland flow) has been modelled in different ways, with significant progress in the development of physics-based deterministic models, based on the mass, energy and momentum conservation laws.
Physically-based mathematical models have been applied using both simplified flow equations and fully dynamic wave equations. Simplified models are based on the kinematic wave approximation or the diffusion wave approximation of the free surface unsteady flow equations, otherwise called the fully dynamic or shallow water equations. In the previous report these simplified models have been analyzed.
In the recent literature, there are a number of papers dealing with the analysis of the overland flow problem using numerical models applied to both ideal case and small real basins. Despite the large interest and the progress in rainfall–runoff modeling, there are very few papers concerning its application to simulate real large scale events. In this context, a literature survey is presented in the next sections.

Venkata R. K., Eldho T. I., Rao E. P, Chithra N. R. (2008). "A Distributed Kinematic Wave–Philip Infiltration Watershed Model Using FEM, GIS and Remotely Sensed Data". ***Water Resour Manage*** **22:737–755.**

The authors show a kinematic wave based distributed watershed model with Philip infiltration model using integrated FEM, GIS and remotely sensed data has been presented here. The developed model has been applied to Banha watershed located in Jharkhand State, India (16.72 km^2). Remotely sensed data has been used to extract the land cover information of the watershed. GIS has been used to prepare the various thematic maps. Finite element grid preparation and generation of input files such as elemental slope and Manning's roughness has been also carried out by GIS. The length of each overland flow element is taken as 250 m and width varies from element to element. The average width of element is ranging from 250 to 530 m.. Sensitivity analysis has been done for the overland Manning roughness also by changing its value by ±10% for all the rainfall events. Peak runoff is more sensitive to Manning roughness followed by volume of runoff. Sensitivity analysis has also been carried out for time step. It is also seen that with decrease in time step, the volume of runoff and peak runoff increased marginally where as the time to peak runoff decreased marginally and vice versa with some minor exceptions.

Ajayi A. E., van de Giesen N., Vlek P. (2008). "A numerical model for simulating Hortonian overland flow on tropical hillslopes with vegetation elements". ***Hydrological Processes*****, 22: 1107–1118.**

This paper describes the development of a two dimensional full hydrodynamic model to simulate overland flow on complex, inclined surfaces with spatially varied soil hydraulic properties. A modified leapfrog scheme with centred time and space derivatives was used to solve the equation. The infiltration component of the model is represented by the Philip-Two-Term equation and the time compression approximation was used to estimate the moment at which rainfall intensity exceeded the infiltration capacity. The rainfall–runoff transformation is dynamic with the computation of infiltration over the duration of rainfall done at every computational node. The model was used to simulate results from a field study in the Kotokosu catchment in the Volta Basin, West Africa. A uniform grid spacing of 10 cm was used, thus the small runoff plot has 400 grid points and the long plot 3600 grids points.

Shih D. S., Li M. H, Wu R. S. (2008). "Distributed Flood Simulations with Coupling Gauge Observations and Radar-rainfall Estimates". ***Water Resour Manage*** **22:843–859**

This study used a two-dimensional diffusion hydrodynamic model for the simulation of extreme flood events that occurred in the Shihmen reservoir watershed (736.4 km^2). The model is an explicit scheme (ADE) applied on various horizontal spacings from 120 to 240 m at an incremental interval of 40 m. Spatial precipitation was provided by the gauge interpolations and weather radar rainfall estimate schemes. A case study, Typhoon Nari on September 16–18, 2001, was then performed using radar-derived rainfall with coupling two-dimensional diffusion hydrodynamic model in flood routing investigations. For each resolution a sensitivity analysis was focused on the Manning's roughness coefficient. Simulation results revealed that decreasing the values of the Manning's roughness coefficient to accelerate the water flow on the coarser grids were a good approach in achieving the minimal error, such as in the case for grid size of 240 m×240 m. The simulated results generated by spatial resolution of 240 m×240 m showed large residual errors compared to the finer grids resolution. The smallest error occurred when the spatial resolution was set as 160 m×160 m and the results only shows a slight difference than that with 120 m×120 m. However, a spatial resolution of 160 m×160 m results in the simulation execution time of about 60% of that required for a resolution of 120 m×120 m. Consequently, the grid size of 160 m×160 m was selected for case study in examining the effect of different spatial precipitation algorithms on distributed flood routing.

Jinkang D., Shunping X., Youpeng X., Xu C., Singh V. P. (2007). "Development and testing of a simple physically-based distributed rainfall-runoff model for storm runoff simulation in humid forested basins". ***Journal of Hydrology*** **336, 334– 346**

The authors discretize the watershed into a number of square grids, which then are classified into overland flow and channel flow elements based on water flow properties. On the overland elements, infiltration, overland flow and lateral subsurface flow are estimated, while on channel flow elements river flow routing is performed. Lateral subsurface flow is calculated using Darcy's law and the continuity equation, whereas overland flow and channel flow are modeled using a one dimensional kinematic wave approximation to the St. Venant equations. The model governing equations are solved by an implicit finite difference scheme. Most of the model parameters can be derived from digital elevation models (DEMs), digital soil and land use data, and the remainder of the parameters, such as Manning's roughness coefficient, that are comparatively sensitive can be determined by model calibration. The model is tested using nine storm events in the Jiaokou watershed, a sub-basin of Yongjiang River in Zhejiang Province, China (259 km^2). The cell size is 100 m. One storm is used for calibrating the model parameters and the remaining eight storms are used to verify he model. When judged by the model efficiency coefficient (R2), volume conservation index (VCI), absolute error of the time to peak (ΔT), and relative error of the peak flow rate (δPmax), acceptable results are achieved. Sensitivity analysis shows that the model is sensitive to saturated hydraulic conductivity (Ks), Manning's roughness coefficients (n) and the initial soil moisture content.

England J. F. Jr., Velleux M. L., Julien P. Y. (2007). "Two-dimensional simulations of extreme floods on a large watershed". ***Journal of Hydrology*** **347, 229– 241**

The authors investigate the applicability of the Two-dimensional, Runoff, Erosion and Export (TREX) model to simulate extreme floods on large watersheds in semi-arid regions in the western United States. Spatially-distributed extreme storm and channel components are implemented so that the TREX model can be applied to this problem. Overland flow is estimated in two dimensions via the continuity equation and the momentum equation using the diffusive wave approximation. TREX is demonstrated via calibration, validation and simulation of extreme storms and floodson the 12.000 km^2 Arkansas River watershed above Pueblo, Colorado. A 960 m grid cell size was used in the modeling overland flow instead a 100 cell area was used to discretize the channel. The parameters that were used to calibrate the model were Manning *n* for overland cells and channel segments, saturated hydraulic conductivity, and initial soil moisture. The model simulates peak, volume and time to peak for the record June 1921 extreme flood calibration and a May 1894 flood validation.

Taskinen A., Bruen M. (2007). "Incremental distributed modelling investigation in a small agricultural catchment: 1. Overland flow with comparison with the unit hydrograph model". ***Hydrological Processes*** **21, 80–91**

A distributed overland flow model is presented and the test results compared with those of the unit hydrograph (UH) model. Infiltration excess in the overland model was calculated using both a modified Green and Ampt (G–A) method and a more complicated method that keeps track of the soil moisture content. The two-dimensional partial differential flow equations with kinematic flow approximation were solved using both backward-central explicit and implicit finite-difference schemes. The model was developed and validated in small agricultural fields in southern Finland, comparisons were made using different storms. The test results showed that the UH models performed as well as the basic distributed model in the calibration, but they could not match the performance of any of the distributed models in verification. The main reason for this was that the assumption of the linearity in the catchment response, essential for the UH models, was not valid. Moreover the implicit scheme was significantly more accurate in the model calibration than the explicit scheme.

Howes D. A., Abrahams A. D., Pitman E. B. (2006). "One - and two-dimensional modelling of overland flow in semiarid shrubland, Jornada basin, New Mexico". ***Hydrological Processes*** **20, 1027–1046.**

Two distributed models, a one-dimensional (1D) model and a two-dimensional (2D) model, are developed to simulate overland flow in two small semiarid shrubland watersheds in the Jornada basin, southern New Mexico. The models represent each watershed by an array of 1 m^2 cells, in which the cell size is approximately equal to the average area of the shrubs. Each model uses only six parameters, for which values are obtained from field surveys and rainfall simulation experiments. In the 1D model, flow volumes through a fixed network are computed by a simple finite-difference solution to the 1D kinematic wave equation. In the 2D model, flow directions and volumes are computed by a second-order predictor–corrector finite-difference solution to the 2D kinematic wave equation, in which flow routing is implicit and may vary in response to flow conditions. The models are compared in terms of the runoff hydrograph and the spatial distribution of runoff. The simulation results suggest that both the 1D and the 2D models have much to offer as tools for the large-scale study of overland flow. The 1D model is better suited to the study of runoff due to individual rainfall events, whereas the 2D model may, with further development, be used to study both runoff and erosion during multiple rainfall events in which the dynamic nature of the terrain becomes an important consideration.

Jain M. K., Singh V. P. (2005). "DEM-based modelling of surface runoff using diffusion wave equation". ***Journal of Hydrology*** **302, 107–126**

A DEM-based overland flow model has been developed for computation of surface runoff from isolated storm events. The model operates on a cell basis and takes cell physical information on topography, land use and soil from a GIS. The catchment DEM is utilized in the model to generate computational flow direction and flow routing sequencing for each of the discretized cell of the catchment. An FVM based numerical solution of the diffusion wave approximation of the St Venant equations for DEM-derived overland and channel flow is developed and found appropriate for modelling of surface runoff from a catchment. The cell-based infiltration is computed using the Philip infiltration model. The proposed model is calibrated and verified, using rainfall and runoff data collected on the Banha catchment in India. The initial values of model parameters were assigned on the basis of land use and soil type present in a cell based on the average values available in the literature. A sensitivity analysis was performed by systematically changing the calibrated model parameter values by -10% and +10%. The model was found to be most sensitive to pore size distribution index, hydraulic roughness, initial soil saturation degree, saturated hydraulic conductivity and effective porosity. It was also found that in comparison to runoff volume, the hydrograph shape and time to peak were affected more by the change in the values of Manning's roughness parameter.

Fiedler F. R., Ramirez J. A. (2000). "A numerical method for simulating discontinuous shallow flow over an infiltrating surface". ***International Journal for Numerical Methods in Fluids*** **(32) : 219–240**

A numerical method based on the MacCormack finite difference scheme for simulating discontinuous shallow flow over an infiltrating surface has been developed. Enhancements to the basic scheme include casting it in fractional steps, treating a stiff source term point implicitly, and upwinding the convective acceleration term. Full dynamic interaction between surface water and infiltration is achieved, where infiltration is modeled with the Green–Ampt equation. It has been shown to be useful for simulating overland flow when spatially variable infiltration and microtopography are important. With proper attention given to the scale of application (determined by the physical process to be simulated and the degree of spatial variability inherent to that process) such that discretization errors are small, the developed method will have several other practical applications. For example, the authors are currently developing a model based on this method to simulate rainfall-driven flash floods, such as the destructive flood that occurred in Fort Collins, Colorado, USA during the summer of 1997.

DEVELOPMENT AND IMPLEMENTATION OF NUMERICAL SCHEME AND TECHNIQUES

As presented above, there are few articles about the simulations of real events of overland flow, most of them, with the exception for Ajayi et al. (2008) and Fiedler & Ramirez (2000), use simplified models based on kinematic or diffusive approximations. Moreover from the previous bibliographical review (Venkata et al., 2008; Shih et al., 2008; Jinkang et al., 2007; England et al., 2007; Jain and Sing, 2005) it is evident that the choice of the roughness's coefficient have a great influence on numerical results, in particular on both the peak discharge and the time to peak, then a sensitivity analysis on the roughness's coefficient will be made.

The goal of this study is to implement a numerical code based on complete 2D shallow water equations.
According to the comparative analysis, widely showed in the previous report, the second order accurate MacCormack's scheme gives best results and it is a relatively simple scheme to discretize both the complete model and the simplified (diffusive and kinematic) models (Gottardi and Venutelli, 2008; Liang et al., 2007; Kazezyılmaz-Alhan and Medina, 2007; Costanzo and Macchione, 2005; Costanzo and Macchione, 2004; Costanzo and Macchione 2002). Thus, in this period, the MacCormack's scheme has been further improved by considering some numerical techniques in order to simulate real overland flow situations.

The implemented code is based on the fully conservative shallow water equations:

$$\frac{\partial \mathbf{U}}{\partial t}+\frac{\partial \mathbf{F}}{\partial x}+\frac{\partial \mathbf{G}}{\partial y}=\mathbf{S} \tag{1}$$

where:

$$\mathbf{U} = \begin{pmatrix} h \\ hu \\ hv \end{pmatrix} \tag{2}$$

$$\mathbf{F} = \begin{pmatrix} hu \\ hu^2 + gh^2/2 \\ huv \end{pmatrix}; \ \mathbf{G} = \begin{pmatrix} hv \\ huv \\ hv^2 + gh^2/2 \end{pmatrix} \tag{3, 4}$$

$$\mathbf{S} = \begin{pmatrix} r - f \\ gh\left(S_{0x} - S_{fx}\right) \\ gh\left(S_{0y} - S_{fy}\right) \end{pmatrix} \tag{5}$$

with:
t is time; x, y are the horizontal coordinates; h is the water depth; u, v are the depth-averaged flow velocity in x- and y- directions; g is the gravitational acceleration; S_{0x}, S_{0y} are the bed slopes in x- and y- directions; S_{fx}, S_{fy} are the friction slopes in x- and y- directions, which can be calculated from Strickler's formula as:

$$S_{fy} = \frac{v\sqrt{u^2 + v^2}}{K_S^2 \ h^{4/3}} \quad S_{fx} = \frac{u\sqrt{u^2 + v^2}}{K_S^2 \ h^{4/3}} \tag{6,7}$$

r is the rain intensity and f are the infiltration losses.

The finite volume method, widely adopted in the literature, has been used to discretize the previous equations. It considers the integral form of the shallow water equations that allows a quite easy implementation of shock capturing schemes on different mesh type.
The system of equation is integrated over an arbitrary control volume $\Omega_{i,j}$ and, in order to obtain surface integrals the application of Green's theorem to each component of the vectors **F** and **G** leads to (Hirsch, 1990)::

$$\frac{\partial}{\partial t} \int_{\Omega_{i,j}} \mathbf{U} d\Omega + \oint_{\partial\Omega_{i,j}} [\mathbf{F},\mathbf{G}] \cdot \mathbf{n} \, dL = \int_{\Omega_{i,j}} \mathbf{S} d\Omega \tag{8}$$

where $\partial\Omega_{i,j}$ being the boundary enclosing $\Omega_{i,j}$, **n** is the unit vector normal and dL is the length of each boundary. Denoting by $\mathbf{U}_{i,j}$ the average value of the flow variables over the control volume $\Omega_{i,j}$ at a given time, the equation (6) may be discretized as:

$$\mathbf{U}_{i,j}^{n+1} = \mathbf{U}_{i,j}^{n} - \frac{\Delta t}{\Omega_{i,j}} \sum_{r=1}^{4} \left[\mathbf{F},\mathbf{G}\right]_r \cdot \mathbf{n}_r \, \Delta L_r + \Delta t \mathbf{S}_{i,j}^{n} \tag{9}$$

The finite volume method, as represented by the equation (8), allows the decomposition of a two dimensional problem into a series of locally one dimensional problems to value the normal flux through every side of a cell. Many algorithms has been proposed for the flux vectors evaluation: the most diffused in literature have been examined and implemented .

In the analysis presented herein the MacCormack second order space centered scheme has been applied.
MacCormack's predictor-corrector scheme has an accuracy of second order in both space and time. The numerical integration of system one is performed in the following form:

$$\mathbf{U}_{i,j}^{p} = \mathbf{U}_{i,j}^{n} - \frac{\Delta t}{\Omega_{i,j}} \sum_{r=1}^{4} [\mathbf{F},\mathbf{G}]_{r}^{n} \cdot \mathbf{n}_{r}\, \Delta L_{r} + \Delta t \mathbf{S}_{i,j}^{n} \tag{10}$$

$$\mathbf{U}_{i,j}^{c} = \mathbf{U}_{i,j}^{n} - \frac{\Delta t}{\Omega_{i,j}} \sum_{r=1}^{4} [\mathbf{F},\mathbf{G}]_{r}^{p} \cdot \mathbf{n}_{r}\, \Delta L_{r} + \Delta t \mathbf{S}_{i,j}^{p} \tag{11}$$

$$\mathbf{U}_{i,j}^{n+1} = \frac{1}{2}\left(\mathbf{U}_{i,j}^{p} + \mathbf{U}_{i,j}^{c}\right) \tag{12}$$

where p and c stand for predictor and corrector values. For each side ($r = 1,\ldots,4$), $\mathbf{F}_{\mathrm{r}}$ and $\mathbf{G}_{\mathrm{r}}$ are obtained referring to upstream and downstream volumes alternately. The sequence is concluded in four time steps.
Generally, second order central schemes introduce spurious oscillations. To avoid such problems, viscosity terms are added to the previous equations in order to prevent non linear instability and to dissipate numerical oscillations.
In literature different procedures have been developed. A very popular algorithms was proposed by Jameson et al. (1981) and applied by Fennema and Chaudhry (1990).
Other studies (Harten et al. 1983) suggest the introduction of suitable correction terms, called limiters, according to which the central schemes assume the Total Variation Diminishing (TVD) properties.

JAMESON ARTIFICIAL VISCOSITY

This technique reduces the numerical oscillations by introducing corrective terms in those regions where high numerical gradients of the hydraulic variables occur, leaving unchanged the others.

These terms are introduced in shallow water equations in following way:

$$\mathbf{U}_{i,j}^{n+1} = \frac{1}{2}(\mathbf{U}_{i,j}^{p} + \mathbf{U}_{i,j}^{c}) + \mathbf{D}(\mathbf{U}) \tag{13}$$

where $\mathbf{D}$ is calculated along spatial directions as:

$$\mathbf{D}(\mathbf{U}) = \mathbf{D}_{x}(\mathbf{U}) + \mathbf{D}_{y}(\mathbf{U}) \tag{14}$$

In particular, starting from the approach suggested by Jameson (1981) for the numerical integration of Euler equations and implemented in the shallow water equations by Fennema and Chaudhry (1990), these terms, replacing the water depth with the surface elevation ($H= z+h$) (Aureli et al., 2000) can be written as:

$$\mathbf{D}_{x}(\mathbf{U}) = \left[\varepsilon_{xi,j+\frac{1}{2}}\left(\mathbf{U}_{i,j+1}^{n} - \mathbf{U}_{i,j}^{n}\right) - \varepsilon_{xi,j-\frac{1}{2}}\left(\mathbf{U}_{i,j}^{n} - \mathbf{U}_{i,j-1}^{n}\right)\right] \tag{15}$$

$$\mathbf{D}_{y}(\mathbf{U}) = \left[\varepsilon_{y_{i+\frac{1}{2},j}}\left(\mathbf{U}_{i+1,j}^{n} - \mathbf{U}_{i,j}^{n}\right) - \varepsilon_{y_{i-\frac{1}{2},j}}\left(\mathbf{U}_{i,j}^{n} - \mathbf{U}_{i-1,j}^{n}\right)\right] \tag{16}$$

The ε values are determined as it is shown for the value of $\varepsilon_{xi,j-\frac{1}{2}}$:

$$\varepsilon_{xi,j-\frac{1}{2}} = \alpha \max(\nu_{xi,j-1};\nu_{xi,j}) \tag{17}$$

where $\upsilon_{xi,j}$ is:

$$\nu_{xi,j} = \frac{\left| H_{i,j+1} - 2H_{i,j} + H_{i,j-1} \right|}{\left| H_{i,j+1} \right| + 2\left| H_{i,j} \right| + \left| H_{i,j-1} \right|} \tag{18}$$

The parameter α in equation (17) is a calibration parameter; Fennema and Chaudhry (1990) suggested a range of value from 0.2 to 3. However, as it will be described in the subsequent sections, a very low value of the α parameter, different from the previous one, has to be chosen to simulate overland flow events.

From equations (18), this operator involves five cells surface elevation variations. The correction value, besides to be proportional to the parameter α, it is also proportional to ν term which is the second derivative of the considered variable, maintaining so, the second order accuracy of the scheme.

This techniques is easy to implement but it is unable to consider the velocities variations and the directions of propagation. Moreover, the arbitrary choice of the parameter α may allow an attenuation of the oscillations but not their complete disappearance.

TVD "Total Variaton Diminishing"

In order to obtain a high resolution extension of MacCormack's scheme and to avoid the growth of numerical oscillations, the term $\mathbf{U}_{i,jn+1}$ is corrected according to TVD theory in the following form (Alcrudo & García-Navarro, 1994):

$$\mathbf{U}_{i,j}^{n+1} = \frac{1}{2}\left(\mathbf{U}_{i,j}^{p} + \mathbf{U}_{i,j}^{c}\right) + \frac{\Delta t}{\Omega_{i,j}}\left[\sum_{r=1}^{4}\left(\mathbf{D}_x \delta y - \mathbf{D}_y \delta x\right)_r\right]^n \tag{19}$$

The added normal flux is expressed as:

$$\mathbf{D}_{x_r}^{n} = \frac{1}{2}\sum_{k=1}^{3}\tilde{\alpha}^k \Psi\left(\tilde{a}^k\right)\left[1 - \lambda\left|\tilde{a}^k\right|\right]\left[1 - \varphi\left(\rho^k\right)\right]\tilde{\mathbf{e}}^k \tag{20}$$

where $\tilde{\alpha}$ is the characteristic variable;

$\tilde{a}$ and $\tilde{\mathbf{e}}$ are the eigenvalues and eigenvectors of approximate Jacobian matrix;

λ is equal to Δ*t*/*d*, where *d* is the distance between neighbouring centroids;

$\Psi(\tilde{a})$ is the entropy correction to the modulus of $\tilde{a}$, thereby avoiding the appearance of non-physical solutions;

$\varphi = \varphi(\rho)$ represents the limiter which allows the TVD condition to be fulfilled in which ρ^k is written as:

$$\rho^k{}_{i,j+1/2} = \frac{\left[\tilde{\alpha}\Psi(\tilde{a})(1-\lambda|\tilde{a}|)\right]^k_{i,j+1/2-s}}{\left[\tilde{\alpha}\Psi(\tilde{a})(1-\lambda|\tilde{a}|)\right]^k_{i,j+1/2}} \tag{21}$$

where:

$$s = sign\left(\tilde{a}^k_{i,j+1/2}\right) \tag{22}$$

In this work the *minmod* limiter is used:

$$\varphi(\rho) = \max\left[0, \min(1, \rho)\right] \tag{23}$$

Stability Criteria

As for all explicit methods, the maximum time step is subject to the stability restriction given by the well-known Courant–Friedrich–Lewy (CFL) condition as it follows:

$$\Delta t = C \frac{\Delta x}{\max(\sqrt{u^2 + v^2} + \sqrt{gh})} \tag{24}$$

where C is the Courant number.

In this context it is necessary to underline that in presence of low values of water depth, as those that generally characterize the rainfall runoff process in the early stage of the phenomenon, the schemes can introduce some numerical anomalies with high velocities values (Esteves et al. 2000; Fiedler & Ramirez, 2000; Howes et at. 2006).
Such effect is more evident in the models based on the complete shallow water equations while it results negligible for the kinematic models as it has already been highlighted in the previous report. For that reason the value of the Courant number used for these simulations is very small ranging from 0.0001 to 0.001.

SOURCE TERM TREATMENT

The treatment of source terms of the shallow water equations is a crucial topic for numerical models. Many authors observed numerical problems and unphysical results in unsteady and steady-state flows. Herein, the source term vector is decomposed in two different parts which are treated separately: the bottom variation $\mathbf{S}_b$ written as:

$$\mathbf{S}_b = \begin{pmatrix} 0 \\ ghS_{0x} \\ ghS_{0y} \end{pmatrix} \tag{25}$$

and the friction term $\mathbf{S}_f$:

$$\mathbf{S}_f = \begin{pmatrix} 0 \\ -ghS_{fx} \\ -ghS_{fy} \end{pmatrix} \tag{26}$$

In MacCormack's scheme a technique proposed by Nujic (1995) has been introduced. According to Nujic spurious oscillations are due to a wrong discretization of the momentum equations and in particular to a numerical incompatibility between the discretization of bottom variations ghS_{0x} (or ghS_{0y}) and the pressure term $gh^2/2$.

To resolve this problem, the author proposed two alternative methods for one dimensional finite difference scheme with the purpose to simulate a phenomenon of steady motion on a rectangular channel with variable bottom.

One of these consists in extracting the term $gh^2/2$ from the flux function **F**, and considering it as a source term. Consequently the free surface variation along the two spatial directions substitutes the bottom variation in the source terms.

So the flux function is modified and the terms:

$$S_{0x} = -\frac{\partial z}{\partial x} \; ; \; S_{0y} = -\frac{\partial z}{\partial y} \tag{27, 28}$$

are replaced with:

$$S_{Hx} = -\frac{\partial H}{\partial x} \; ; \; S_{Hy} = -\frac{\partial H}{\partial y} \tag{29, 30}$$

where $H = z+h$ is water elevation and z is bed elevation (Costanzo e Macchione, 2005; Costanzo e Macchione, 2004; Costanzo et al., 2002).

In the present work free surface slope is discretized in a forward and backward manner according to the flux terms.

As for the friction term of the momentum equations and in particular for equations (6-7), the presence of the water depth in the denominator disallows zero depths.
However, a singularity can still arise with regard to the bed friction term when the water depth becomes very small as it is often present in overland flow simulations. For small water depths, the bed friction term dominates over other terms in the momentum equation, as the term $K^2h^{4/3}$ appears in the denominator.
Few previous works have treated in detail the consequences of the discretization of the friction term (Burguete et al. 2007, Liang et al., 2007). The most commonly and simple reported procedure is the pointwise discretization of the terms independent of the methodology used for the rest of the system. However this criteria in presence of high friction or low water depth leads to numerical instabilities. Therefore the time step reducing over the minimum value given by the CFL condition, or an implicit treatment of the pointwise discretized friction source term are needed (Liang et al., 2007; Costanzo and Macchione, 2006, Yoon & Kang, 2004; Caleffi et al., 2003). In this work this is a very evident problem due to the low values of water depths. According to Burguete et al. (2007), the technique based on the limitation of the friction term's values has been implemented in order to avoid incorrect values of the friction term in unsteady cases of advancing front over dry and rough surfaces. Given that the maximum effect of the friction force is to stop the water flow, a necessary condition in the solution is that the updated value of the unit discharge along the two spatial directions (hu) and (hv) at $n+1$ time step after the addition of the discrete friction term retains the same sign of the value at the previous time level n. Then when the numerical friction force exceeds the sum of the other terms in momentum equations that leads to change the sign of the velocity, it will be limited to the maximum value and the velocity is set to zero. Moreover for small water depths also an implicit approach to discretize friction terms is used.

Wet/dry front

In some research studies, the complex wetting/drying phenomenon is simulated by imposing a thin layer of water on dry cells. In this way, the computation is always carried out everywhere regardless of the wet/dry condition. However, this simple treatment is not enough for overland flow situations where water level gradients over relatively steep dry grounds occur inducing unreasonably large velocities. Then, in this work, specific treatments for calculating wet/dry fronts are also applied.

Boundary conditions

In this work the Riemann solver theory is used in the description of outflow, and wall boundary conditions. The sufficient conditions imposed at the boundaries combined with equations obtained from characteristics theory give the information needed for the calculation of boundary flux.
In the applications here presented the outflow boundary conditions have been calculated according to the Riemann theory which implies the constant value of $u+2c$ along a characteristic line. Then, according also to the flow regime, the imposed values at the boundary (B), are computed using the value of the neighbour cell (L) as follows:

- *Sub-critical outflow*: The tangential velocity v and another variable must be imposed. When the water depth h is imposed the normal velocity u is determined as:

$$u_B^{n+1} = u_L^n + 2\sqrt{g}\left(\sqrt{h_L} - \sqrt{h_B}\right) \tag{31}$$

  When normal velocity u is imposed the water depth h is determined as

$$h_B^{n+1} = \left(\left(u_L^n - u_B^n\right)/2g + \sqrt{h_L}\right)^2 \tag{32}$$

  In the analyzed applications generally the imposed variable is determined considering a critical flow regime at boundary.
- *Supercritical outflow:* None of the variables must be imposed:

$$u_B = u_L,\ v_B = v_L,\ h_B = h_L \tag{33}$$

Applications

The model developed by the laboratory LAMPIT (Department of Soil Defence - University of Calabria), described above, has been applied to simulate overland flow event on real topography.
The study of overland flow processes in real situation often refers to a wide area; as a consequence, in order to avoid a significant increase in terms of both computational times and memory storage, the computational domain is divided by using very coarse cells.
In this context, in each application, an analysis on the accuracy of the numerical solutions in relationship to the size of computational cell has been performed. Therefore the effects of

meshing size on numerical results were also analyzed. Finally the implemented code was applied to simulate the surface runoff over a part of Esaro Basin (Calabria).

A first application (**Test 1**) regards the simulation of surface runoff on a simple catchment. The domain is 160 m x 350 m. The elevation data of the watershed has been obtained from a digitized map. The figures 1 and 2 show the surface elevation and the contour lines. In order to compare the effect of the spatial resolution on numerical results the domain has been divided using a 0.5, 1, 2, 3, 5, 10 and 15 m grid size. For all computations the Stickler's coefficient has been imposed equal to 8 $m^{1/3}/s$. Two different constant in space and time precipitation intensities (10 mm/h and 100 mm/h) have been considered over all the domain for a duration of 30 min; the infiltration rate was set equal to zero. In this case the Jameson artificial viscosity has been used to avoid the onset of numerical oscillations. For each grid size the value of the α parameter has been properly chosen in order to fulfill the mass conservation property. The above written boundary conditions are considered.

Excessively fine grids, such as 0.5 m×0.5 m, 1 m×1 m and 2 m x 2m, have been found not to be suitable for the simulations because of restrictions on computer time and memory storage. Instead, the spatial resolutions ranging from 3 m×3 m to 15 m×15 m were set for investigations.

In figure 3 the flow path after 30 min for a constant precipitation intensity equal to 100 mm/h are reported while figure 4 shows the discharge hydrographs at the channel outlet computed using different mesh size (3 m, 5 m, 10 m, 15 m). As it is possible to notice the four hydrographs present little differences due to the diffusive effect of the coarse mesh size.

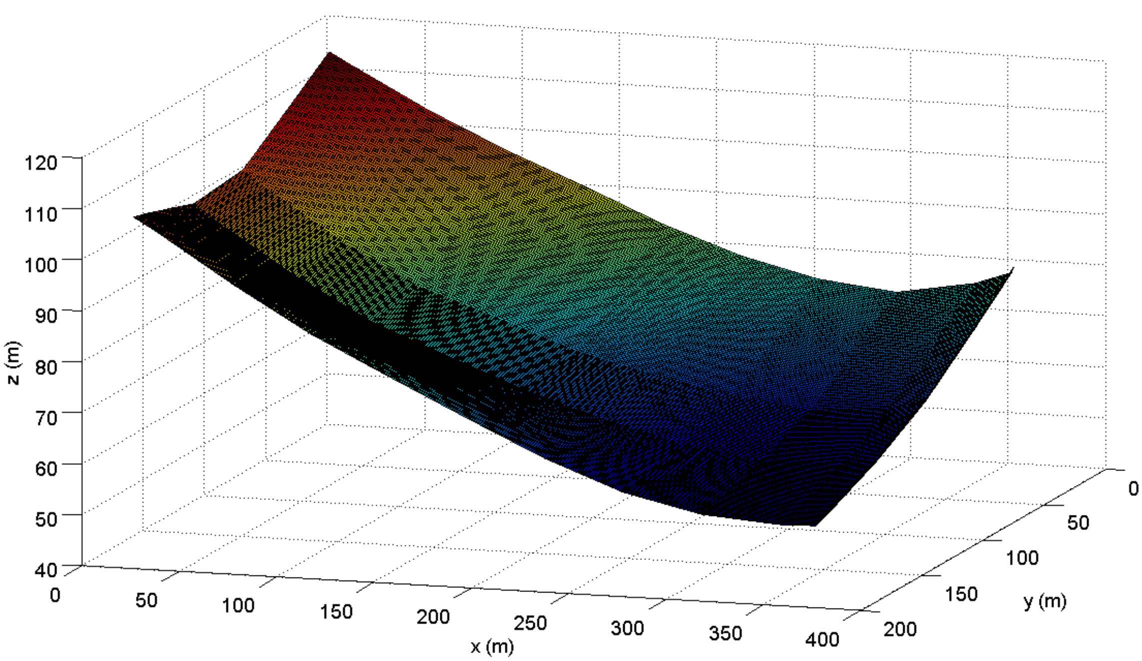

Figure 1. Test 1 – Surface elevation

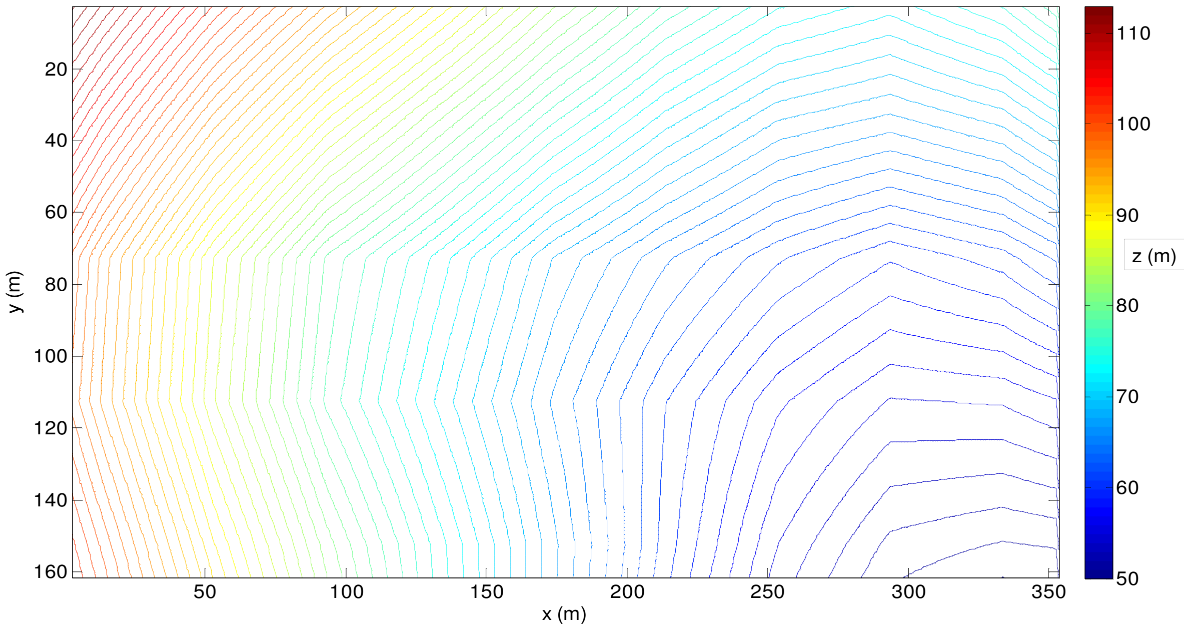

Figure 2. Test 1 – Contour lines

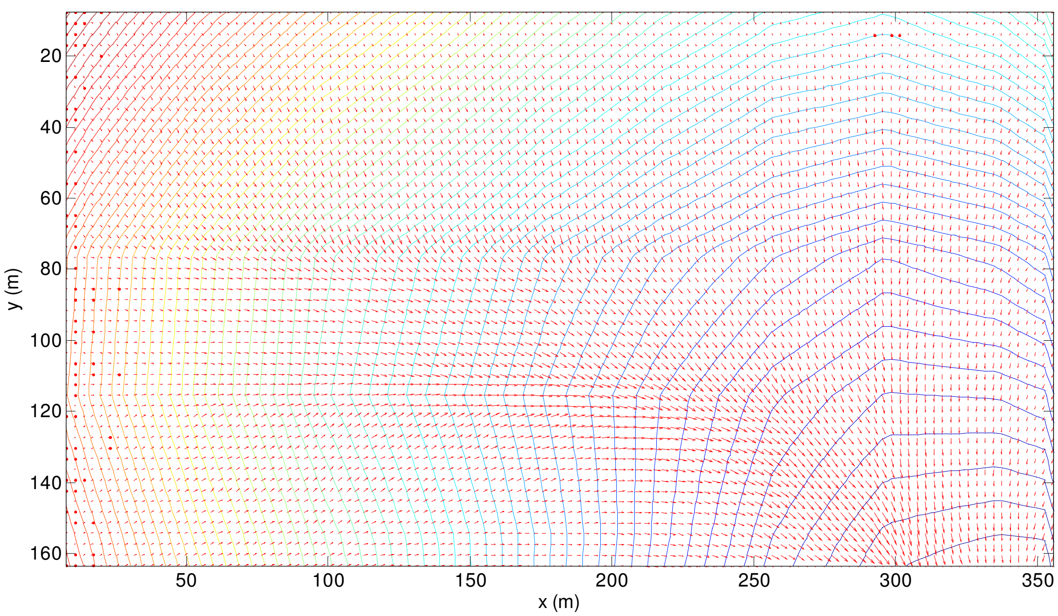

Figure 3. Test 1 – Flow path at t = 30 min

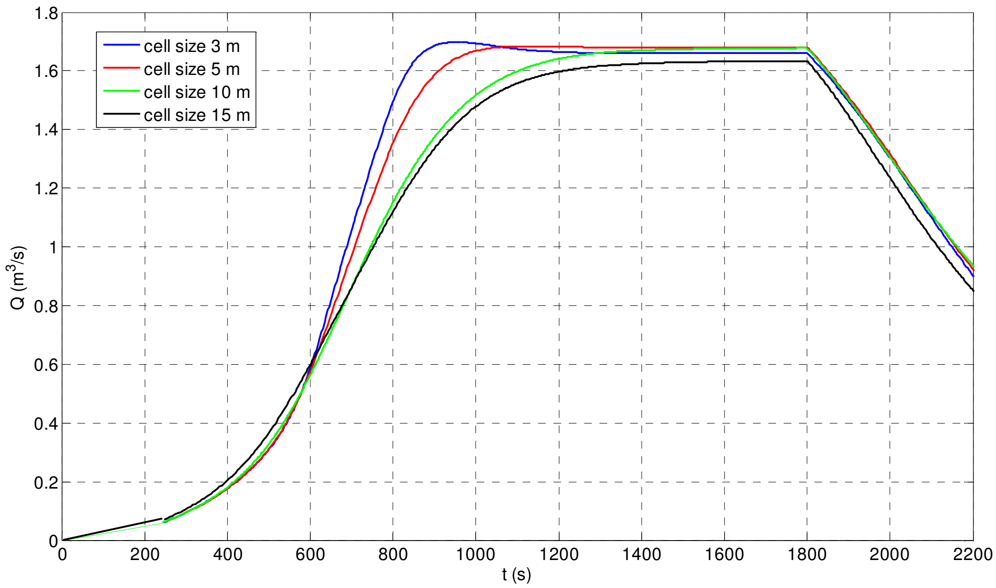

Figure 4. Test 1 – Discharge Hydrographs at the channel outlet computed using different mesh size (3 m, 5 m, 10 m, 15 m) with 100 mm/h rainfall intensity

The simulation for a 10 mm/h rainfall intensity is very difficult due to the presence of very small water depths. In this case it is necessary to impose in equation (24) a very low value of the Courant number in order to avoid numerical instabilities; as a consequence, the simulation clearly requires long computational time and high memory storage. The discharge hydrographs at the channel outlet simulated using 5 m and 10 m grid size are reported in the figure 5. The differences between the simulations are rather little.

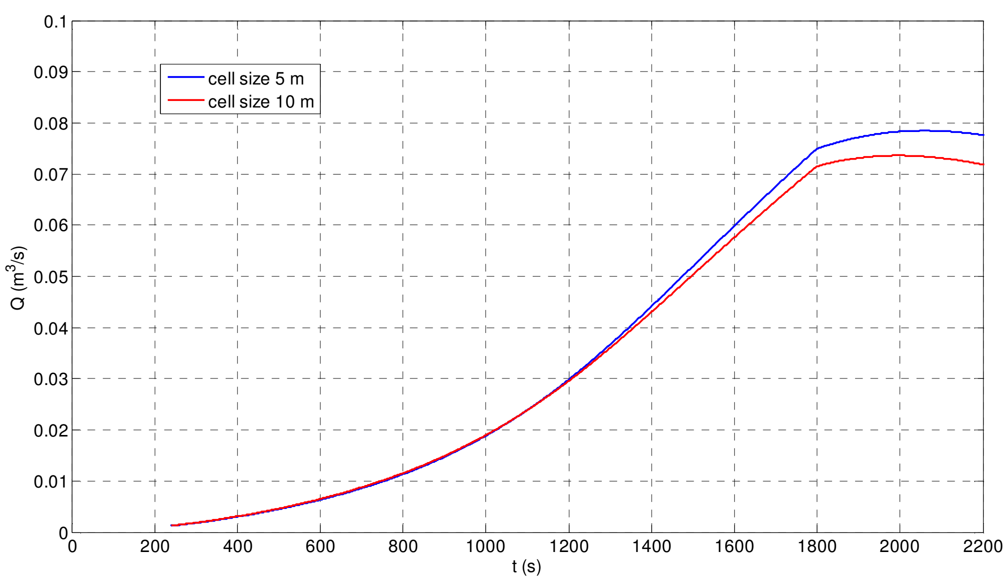

Figure 5. Test 1 – Discharge Hydrographs at the channel outlet computed using different mesh size (5 m, 10 m) with a 10 mm/h rainfall intensity

The second case (**Test 2**) regards a more complex watershed. The domain is 950 m x 1100 m. The elevation data of the watershed have been obtained also from a digitized map. The figures 6 and 7 show the surface elevation and the contour lines.

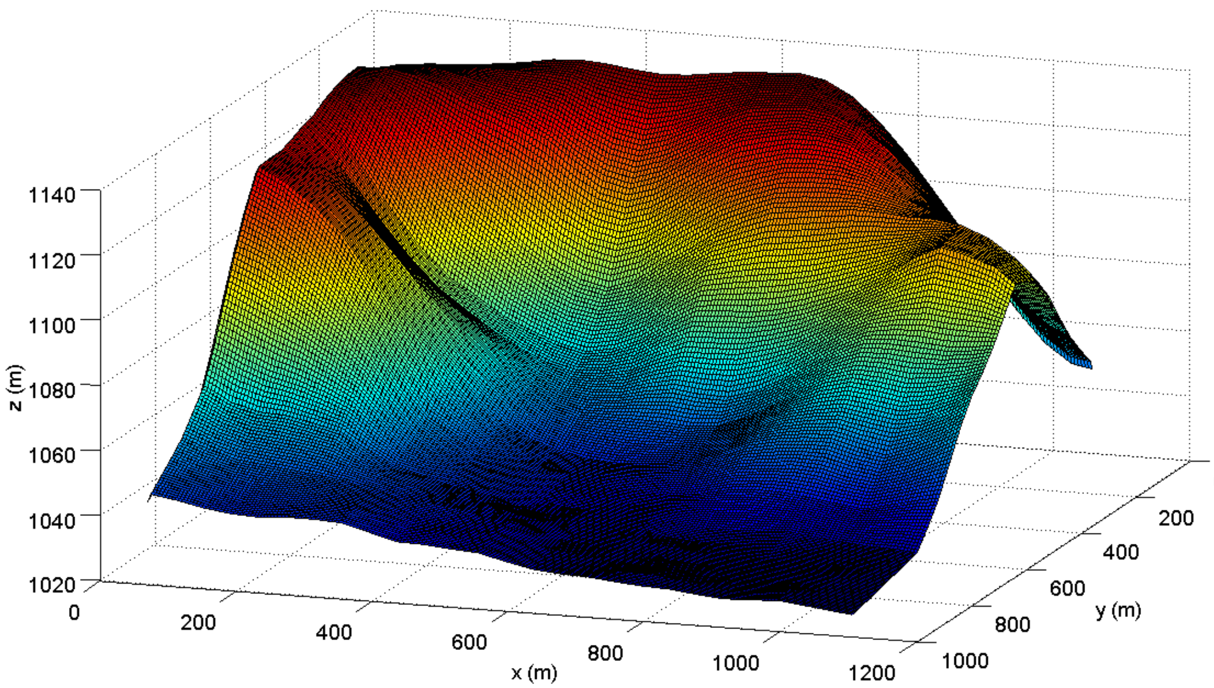

Figure 6. Test 2 – Surface elevation

The domain has been divided according to a structured grid with different cell size (5, 20, and 40 m) and an analysis of the cell size influence on the numerical results has been performed. Also in this case the simulations refer to the propagation of surface runoff due to a constant in time and space rainfall intensity (100 mm/h and 10 mm/h). The Strickler's coefficient is constant in all domain (8 $m^{1/3}/s$) and the infiltration rate is set to zero.
In the figures 8 - 13 water depths and the flow vectors at different time (15, 30 and 45 min) for a 100 mm/h rainfall intensity are reported.

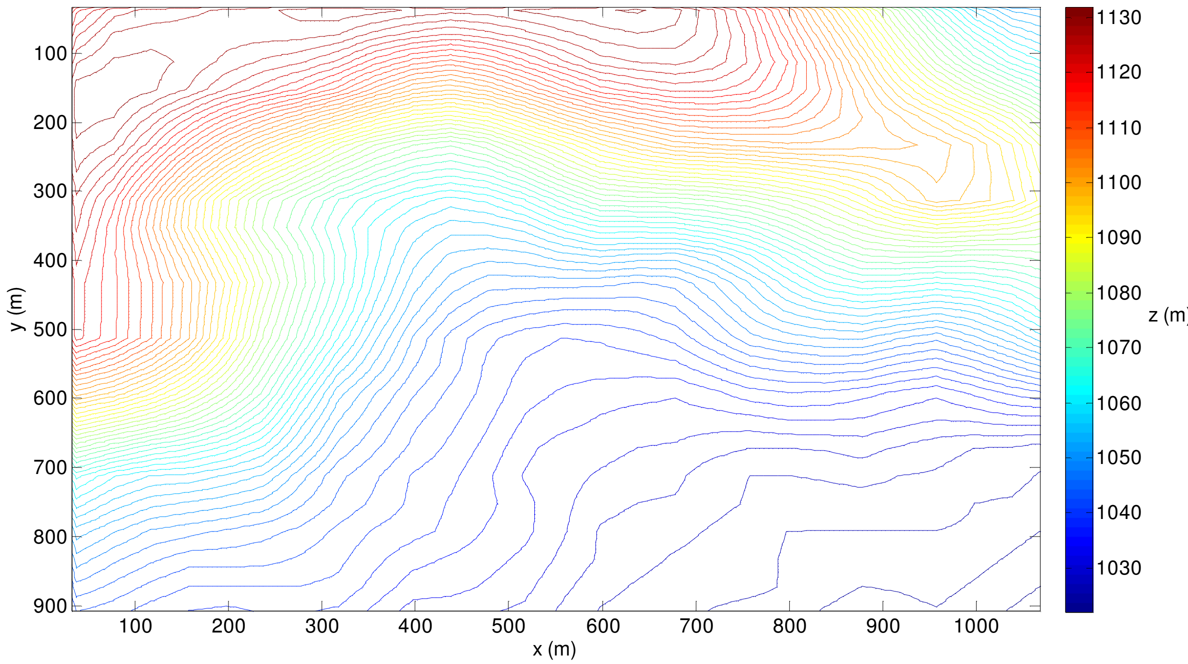

Figure 7. Test 2 – Contour lines

The coarser is the cell size, the smaller is the value to set for Courant number. In particular C = 0.001, 0.0006, 0.0001 were set for $\Delta x = \Delta y$ = 5 m, 20 m, 40 m respectively.
Moreover the high irregular topography has required the application of the TVD technique in the SWE in order to avoid numerical oscillations. This technique, more difficult to implement than Jameson limiter, has the advantage to be more robust and efficient for simulating thin water on irregular topography and it not presents any parameters evaluation. Indeed the choice of the parameter α significantly influences the mass conservation when Jameson limiter has been used in first application of the implemented code. In the figure 14, the mesh size influence on the discharge hydrographs, relative to a 100 mm/h rainfall intensity, at the domain outlet is shown. In the figure 14 it is possible to observe that the mesh size mainly influences the peak discharge while a less variation may be noted in the time to peak values. However the hydrographs are similar showing little differences among the simulations.
The same analysis has been performed to simulate the surface runoff due to a 10 mm/h rainfall intensity. It should be born in mind that the simulation of this situation is the most difficult one due to the presence, for the whole time, of very little water depths that induce numerical instabilities. A robust wet dry procedure, described above, has been thus implemented. The obtained discharge hydrographs are shown in figure 15. In this case the computed hydrographs are very similar. Moreover in all simulations the mass conservation property is fulfilled.

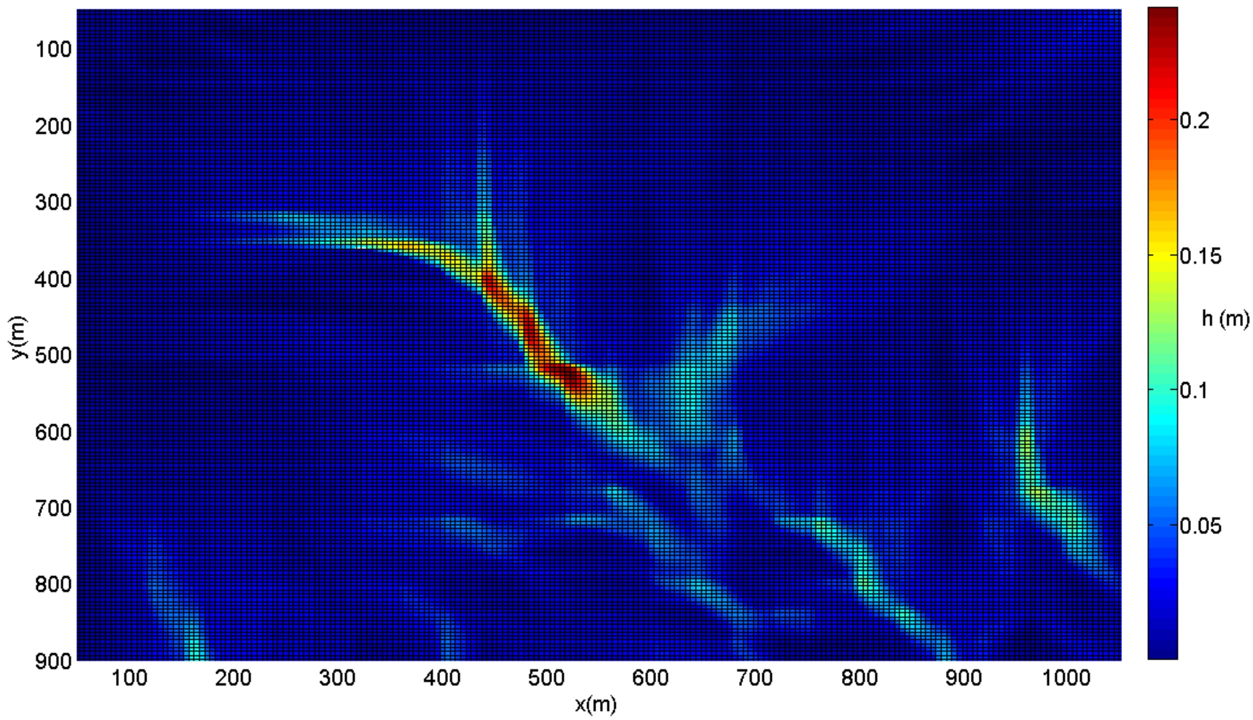

Figure 8. Test 2 – Water depths at *t* = 15 min

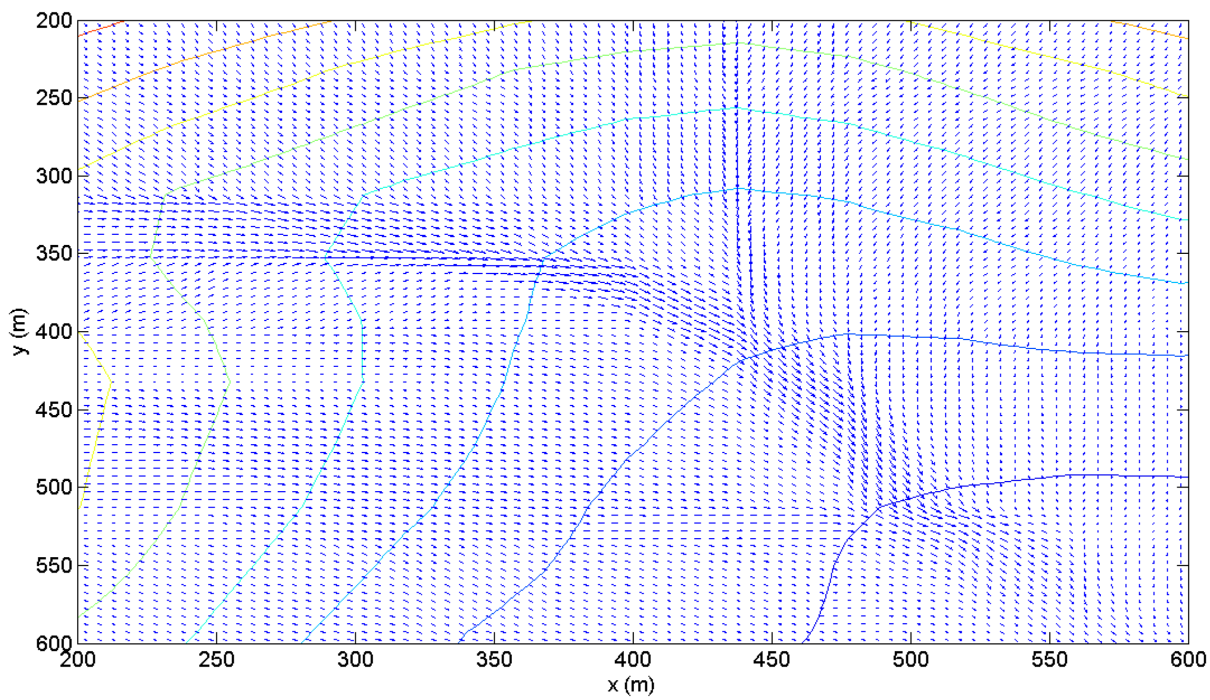

Figure 9. Test 2 – Flow path at *t* = 15 min

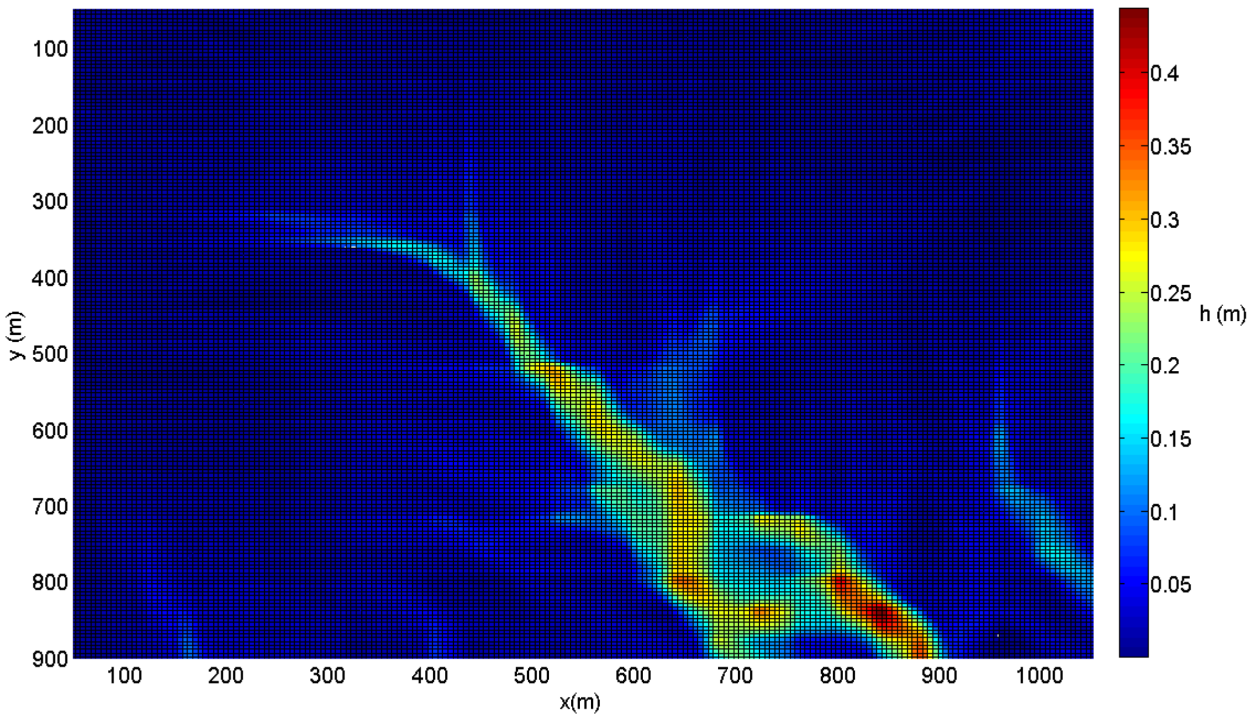

Figure 10. Test 2 – Water depths at t = 30 min

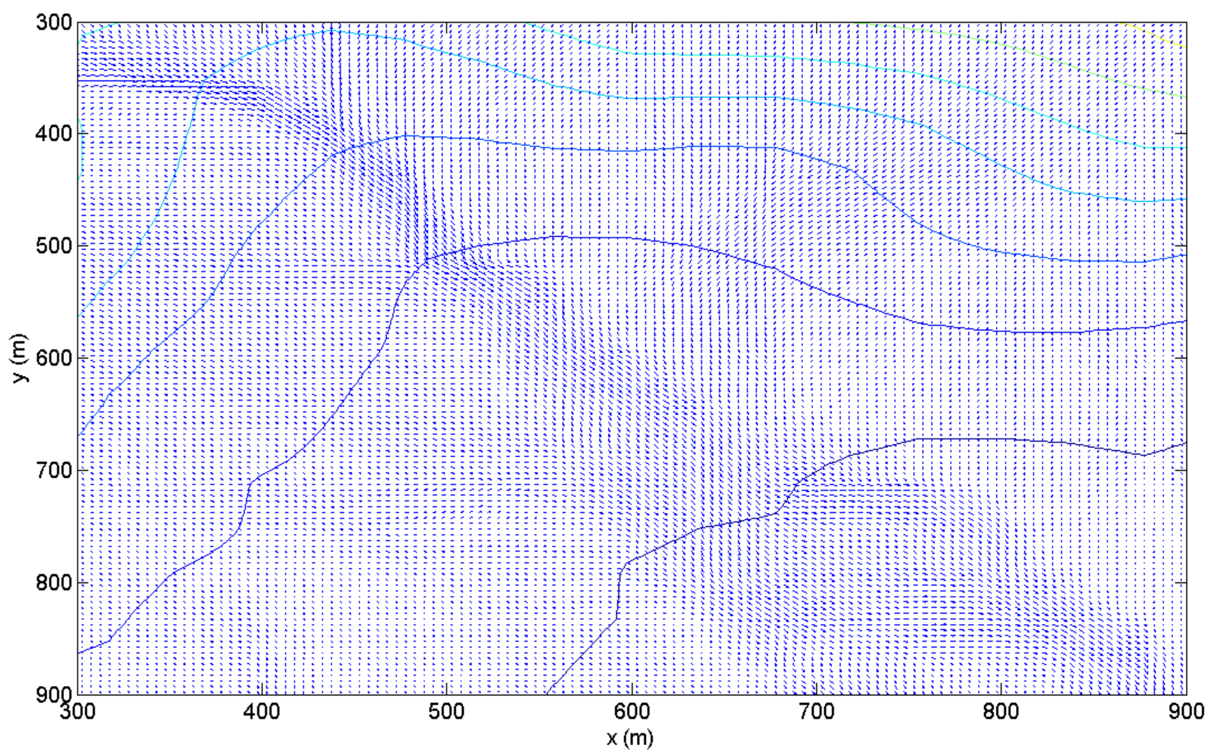

Figure 11. Test 2 – Flow path at t = 30 min

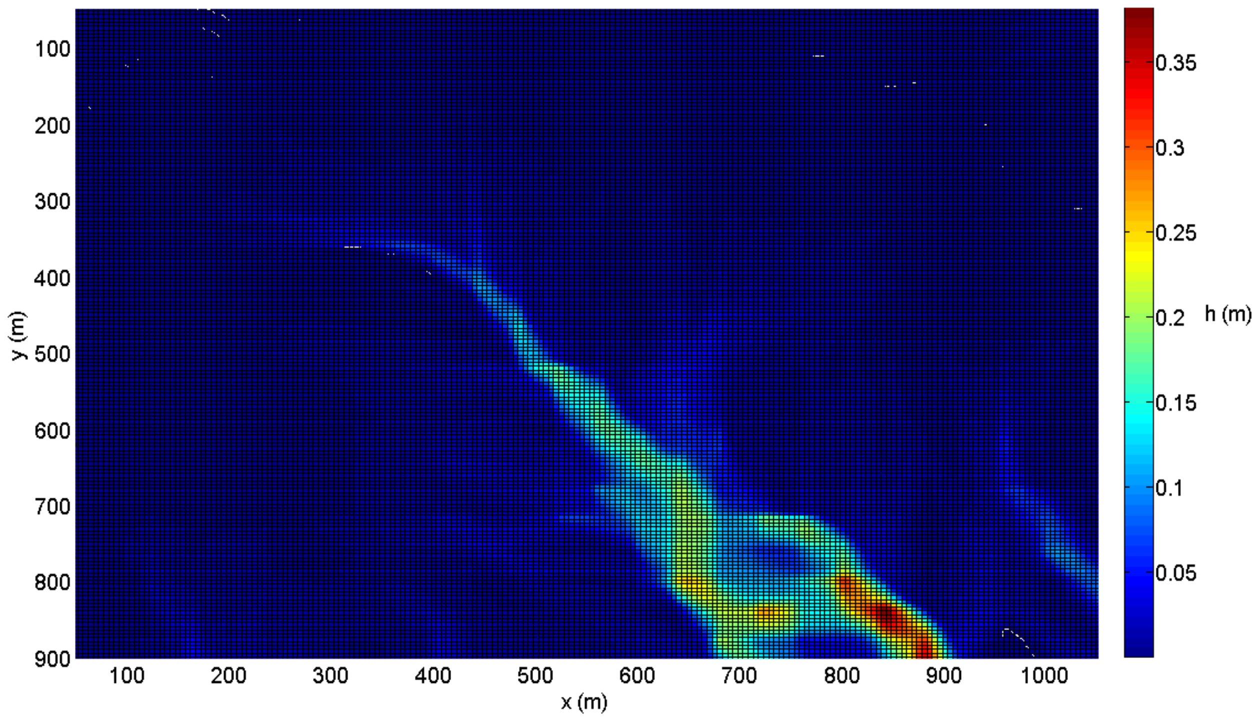

Figure 12. Test 2 – Water depths at *t* = 45 min

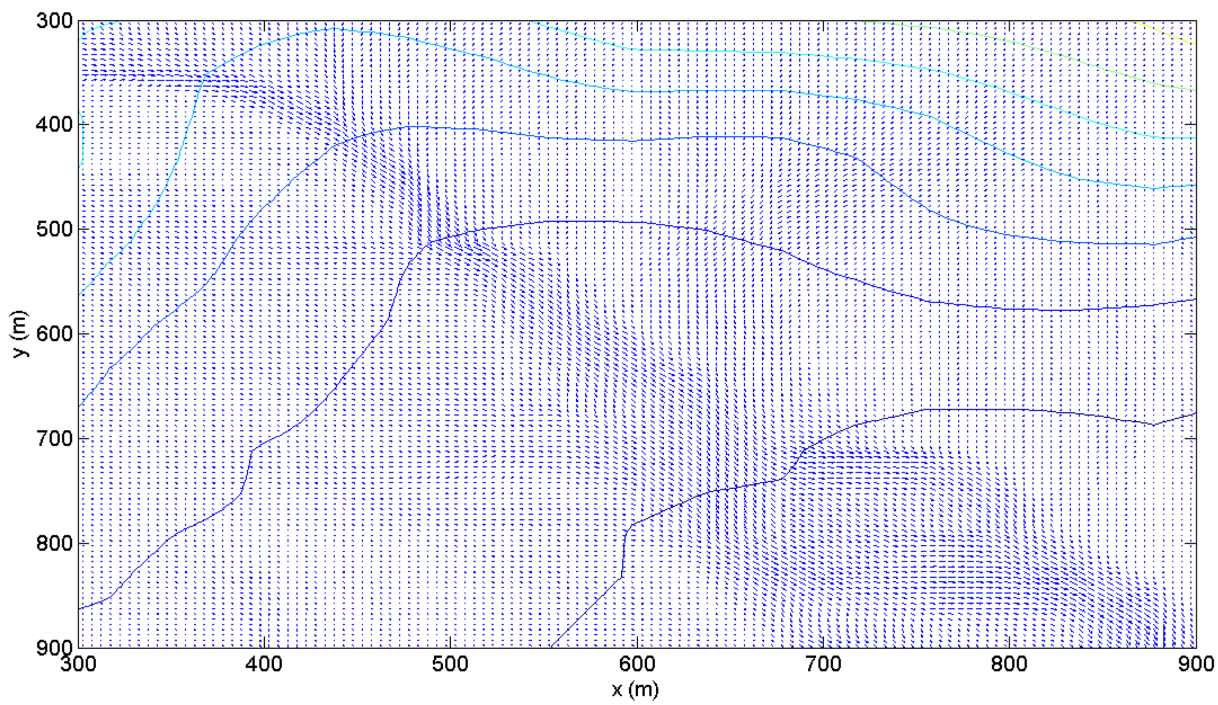

Figure 13. Test 2 – Flow path at *t* = 45 min

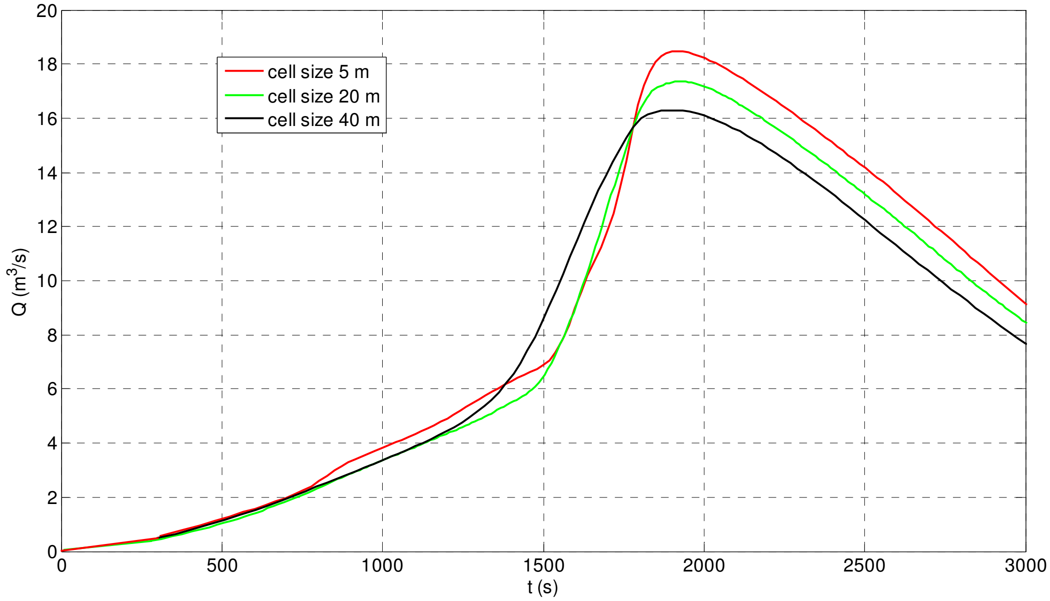

Figure 14. Test 2 – Discharge Hydrographs at the channel outlet computed using mesh size (5 m, 20 m, 40 m) with 100 mm/h rainfall intensity

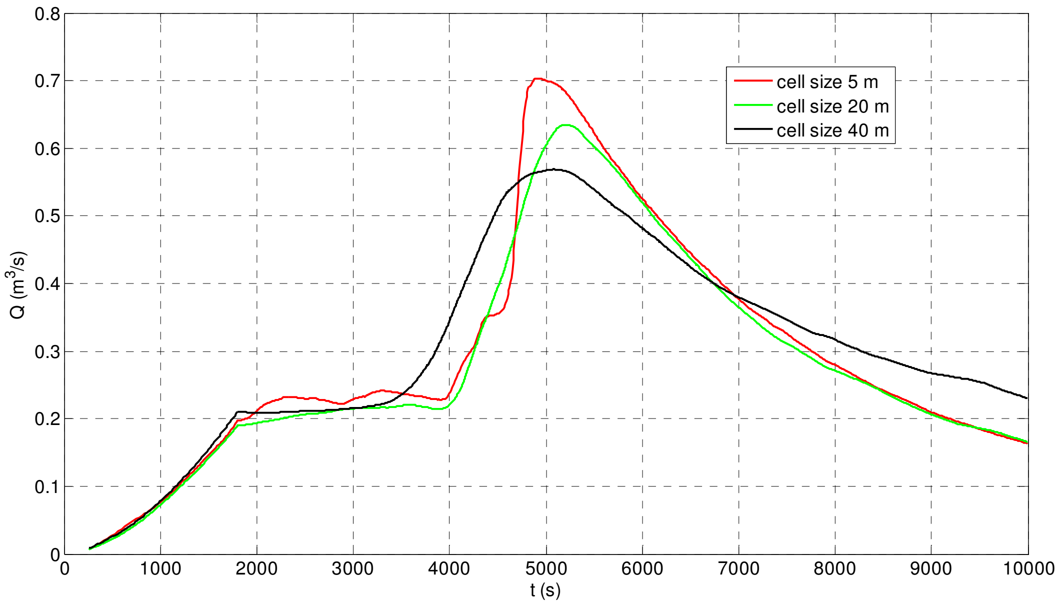

Figure 15. Test 2 – Discharge Hydrographs at the channel outlet computed using different mesh size (5 m, 20 m, 40 m) with 10 mm/h rainfall intensity

The third application (**Test 3**) refers to the simulation of an overland flow phenomenon over the Esaro basin (Calabria).

The morphological description of the Esaro basin has been carried out by means of a GIS software. A lot of problems have been encountered during the DEM generation of Esaro basin mainly due to the cartographic original data whose quality was sometimes not clear at all.

The pre-processing activity had the purpose to determine the input data for the model of propagation of the flood. Particularly it concerned the definition of the digital elevation model of the considered domain. The digital elevation model was obtained from digital maps. It was used a 1:4000 scale map in the system Gauss-Boaga covering the Crotone territory. These data were processed using the GIS software and a DEM with a resolution of 5 m was obtained. Particularly, among the different methods of spatial interpolation provided by the software, a procedure suggested by Hutchinson (1988) has been used, which permits to obtain a well-connected drainage catchment and a correct representation of the river thalwegs. Moreover, during this work, particular attention has been given to the edges management for each cartographic map sheets in order to avoid an incorrect interpolation near the edges themselves. Through the GIS software from the obtained DEM it was possible to delimit the basin at the closing section of St. Francis Bridge. The basin area is equal to 86 km^2.

An aggregated DEM with resolution of 40 m has been used as data input in the implemented code. The result is a discretization of the domain with a structured grid of 410 x 250 cells.

Figure 16 shows the basin and the closing section while figures 17 and 18 show the generated DEM and the computational domain with structured mesh of 40 m cell size.

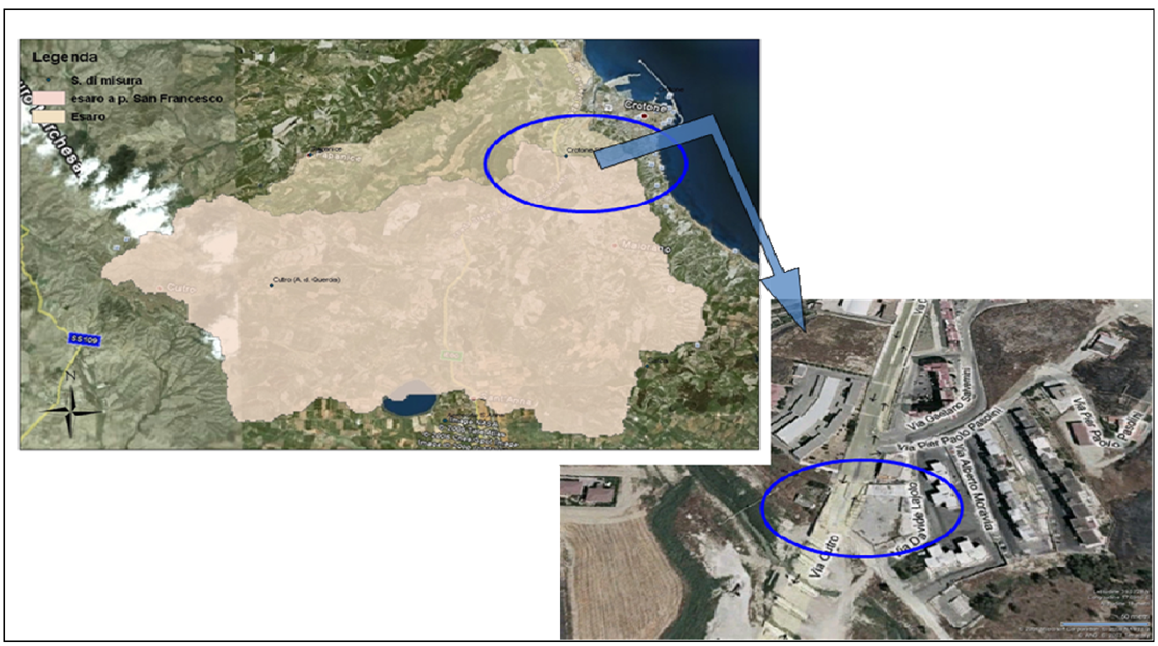

Figure 16. Test 3 – Esaro Basin and the closing section

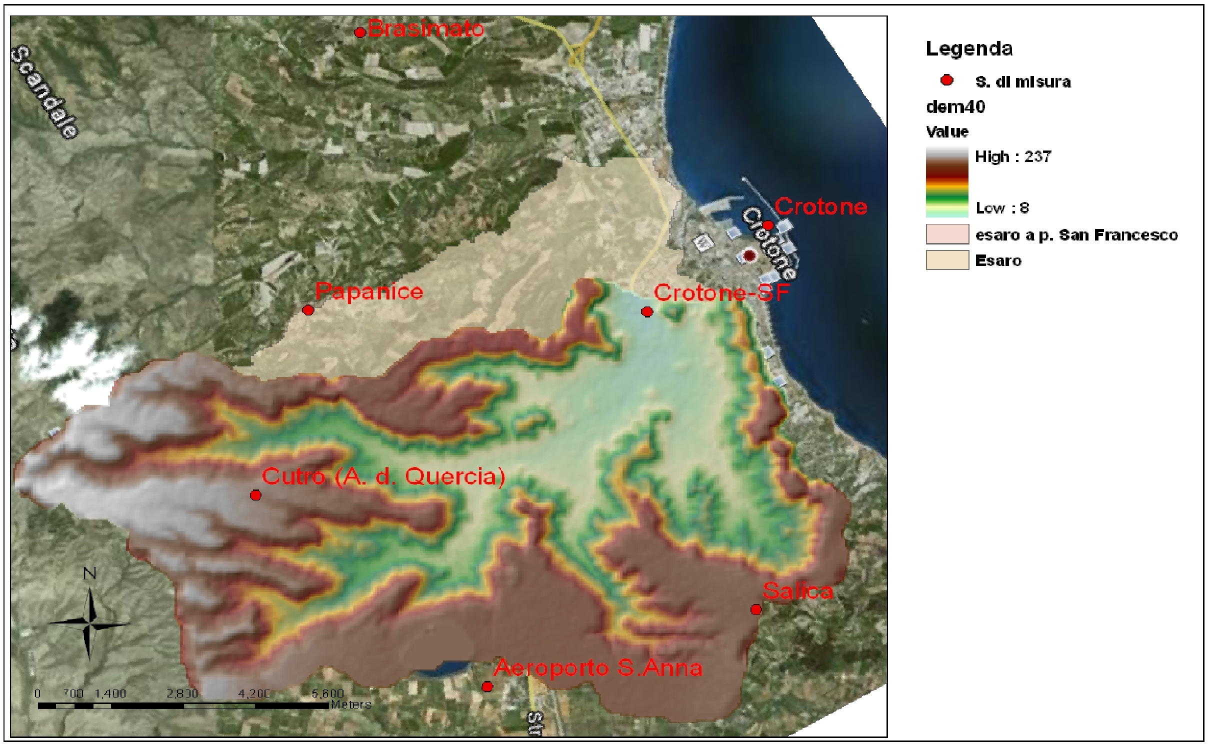

Figure 17. Test 3 – DEM of the Esaro Basin

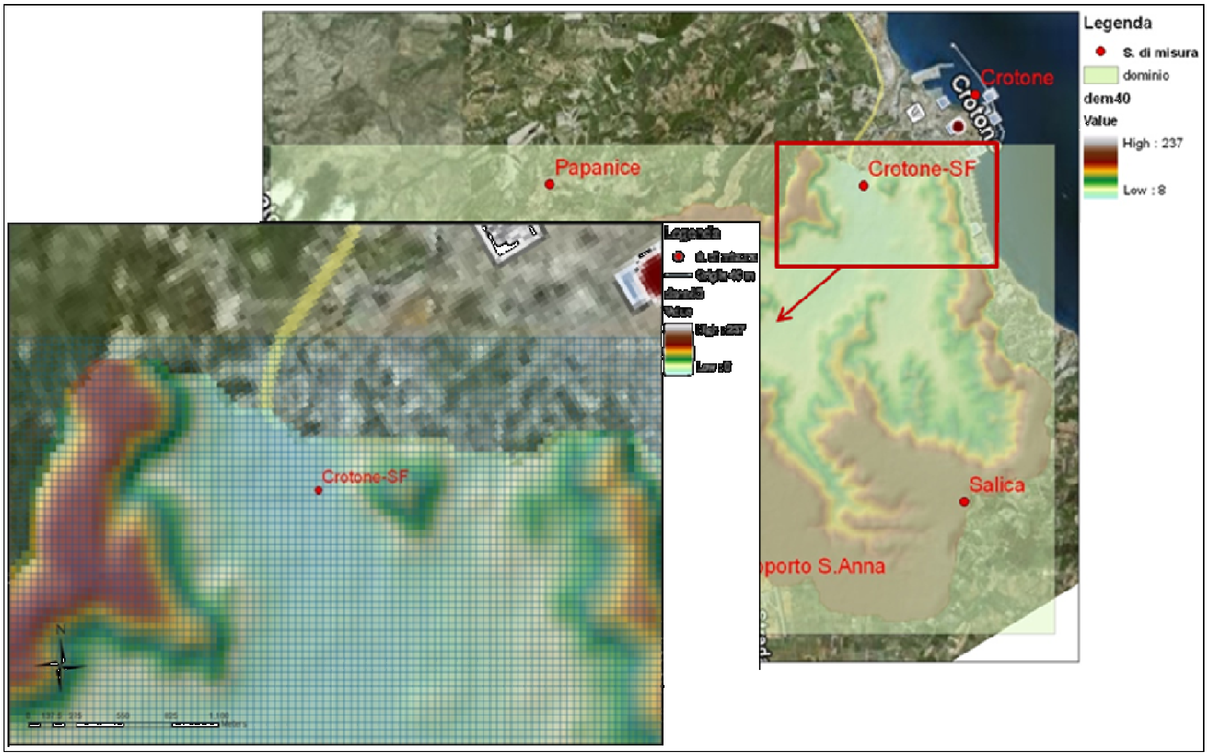

Figure 18. Test 3 – Computational domain using a structured mesh on Esaro Basin

Overland flow simulations in Esaro basin has been revealed very complex due to both the large extension and the presence of an high irregular topography. In order to investigate in deep the causes of numerical anomalies, we have focused the attention on a part of the basin in which numerical instabilities and other similar problems may occur because of the irregularity of the surface elevation.
The analyzed domain is 2000 m x 2000 m wide and its surface elevation and contour lines are shown in figures 19 and 20. The mesh grid has a 40 m cell size.
A constant rainfall (100 mm/h) has been considered over the domain. In this simulation the TVD terms have been also introduced in the shallow water equations.
In the early stage of the simulation. the presence of thin water depths over an irregular topography with high slope, hollows and obstacles to the flow, have induced high values of velocities that progressively leads to the code failure.
Efficient wet dry conditions have been thus implemented together with the particular treatments of the source terms, described in the previous chapter, to prevent the aforementioned problems.
In particular, a water depth of 0.001 m is considered as the inferior depth limit below of which the velocities are set to zero.
In the figures 21 - 30 the numerical results in terms of water depths and flow paths at different time are shown.

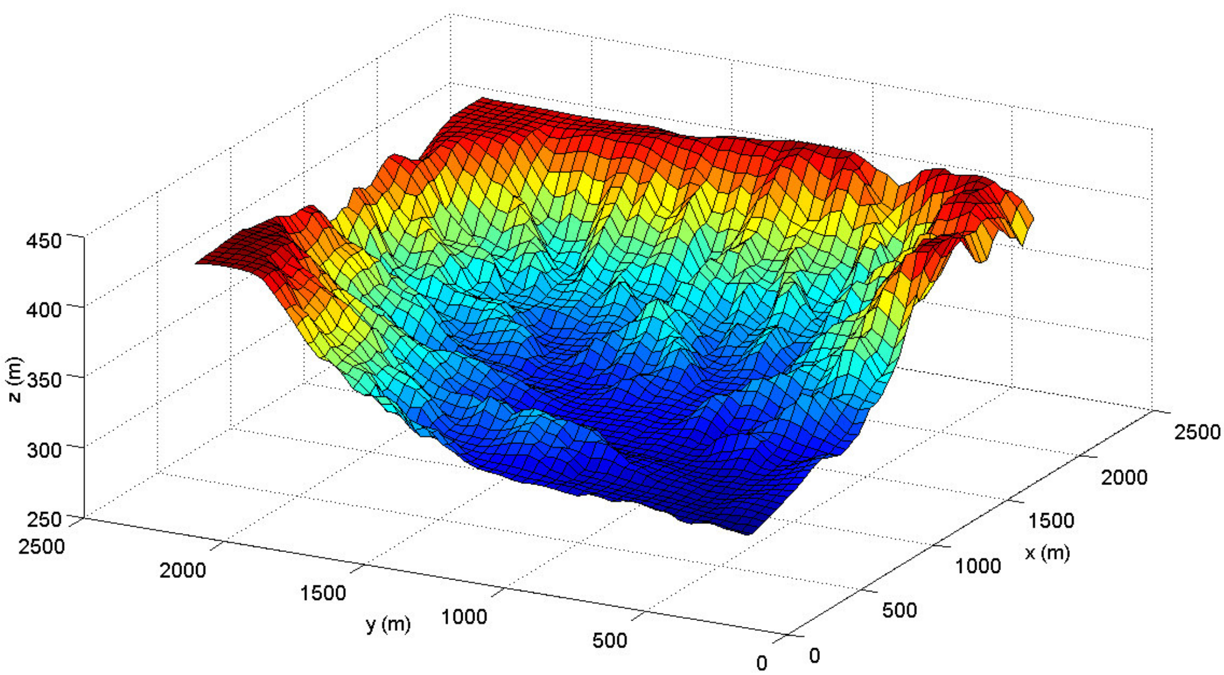

Figure 19. Test 3 – Surface elevation

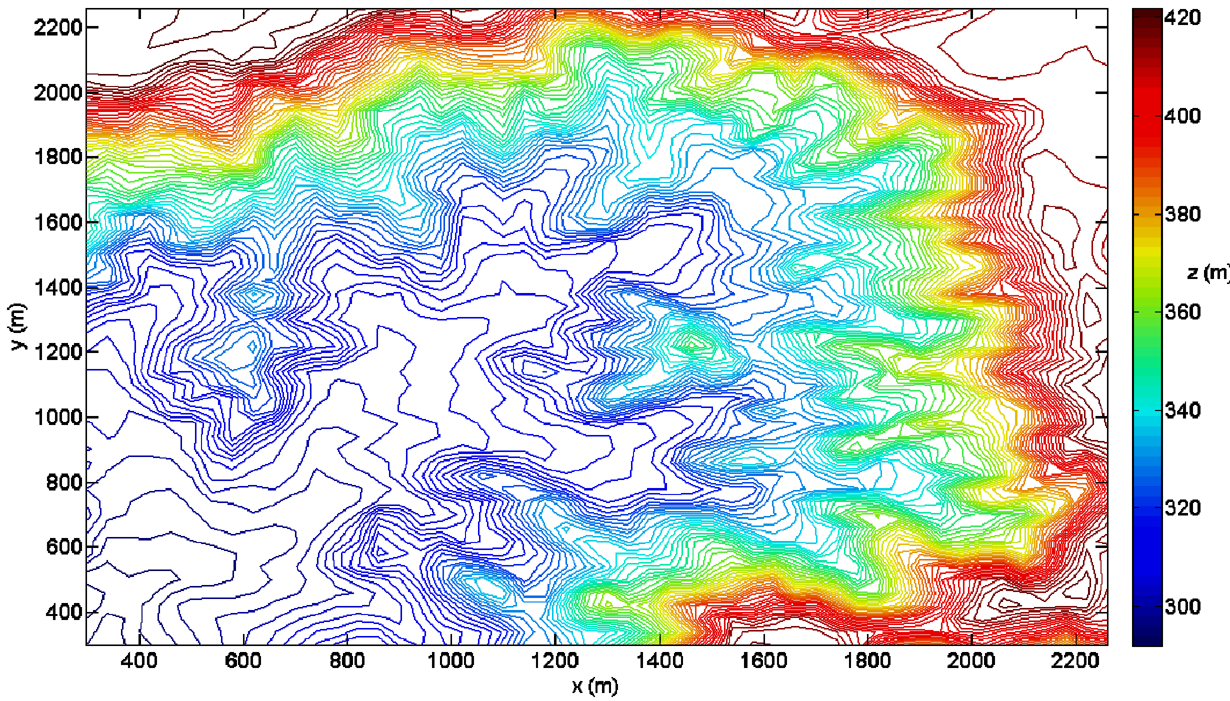

Figure 20. Test 3 – Contour lines

A sensibility analysis of Strickler's coefficient has been also performed for this case. Values of 5, 10, 15 and 25 $m^{1/3}/s$ have been considered as constant friction coefficient in the whole domain. In figure 31 the discharge hydrographs at the outlet of domain, obtained using different Strickler's coefficients, are shown. The phenomenon has been simulated until the hydrographs reach their time to peak because a constant in time and space rainfall intensity (100 mm/h) has been considered. Moreover from figure 31 it is possible to note that the choice of the Strickler's coefficient have a great influence on both the peak discharge and the time to peak. These same conclusions are reported in other papers presented in literature (Venkata et al., 2008; Shih et al., 2008; Jinkang et al., 2007; England et al., 2007; Jain and Sing, 2005).

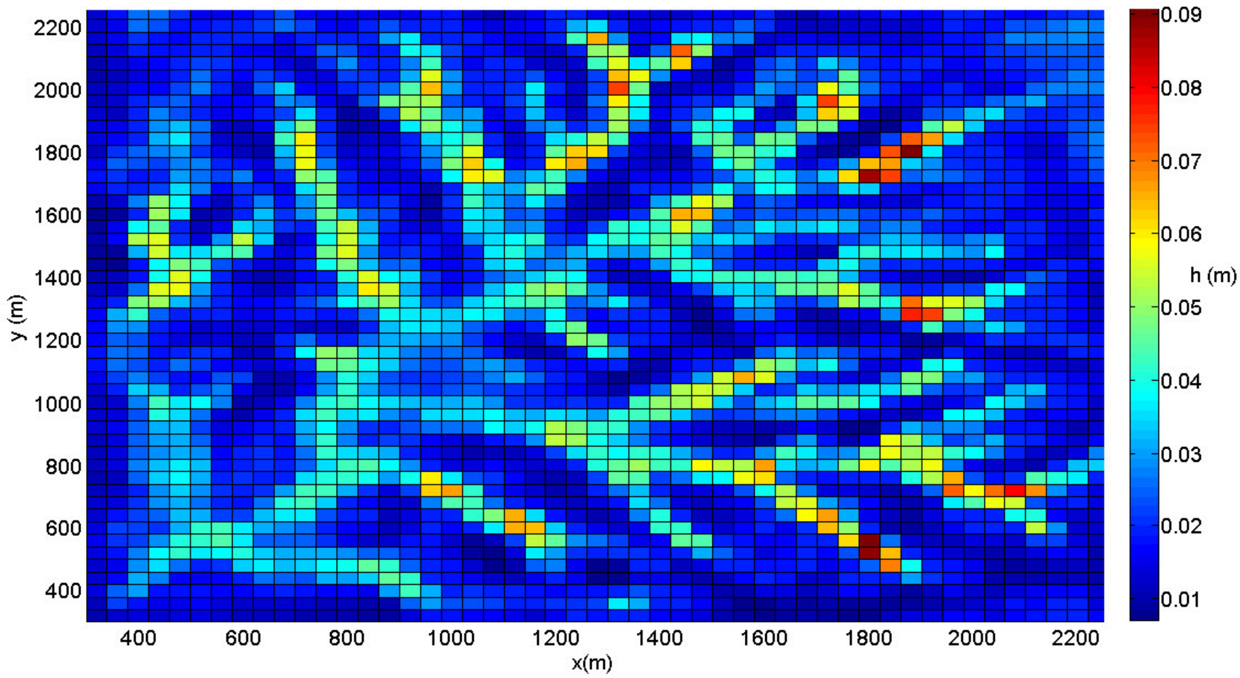

Figure 21. Test 3 – Water depths at t = 15 min

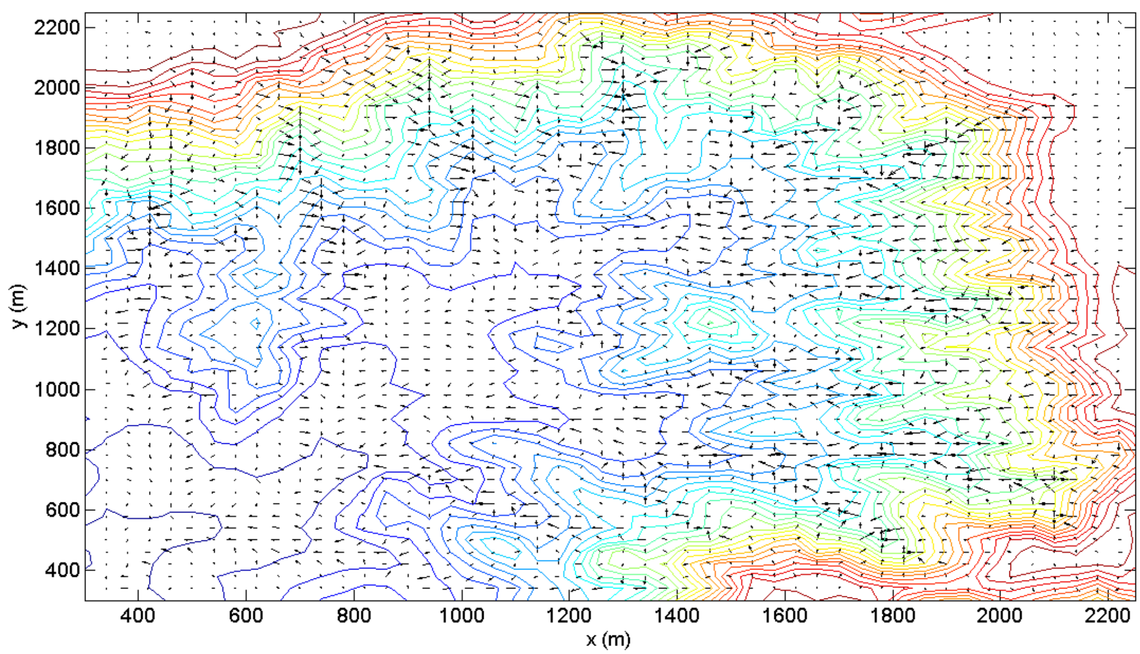

Figure 22. Test 3 – Flow path at t = 15 min

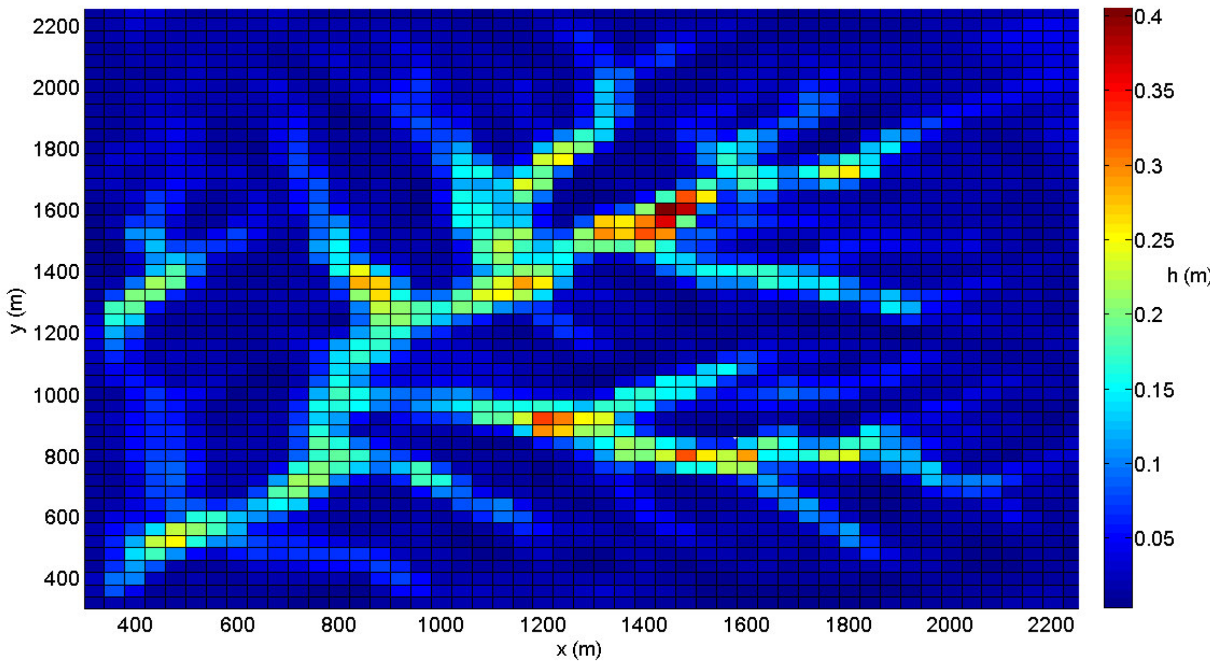

Figure 23. Test 3 – Water depths at t = 30 min

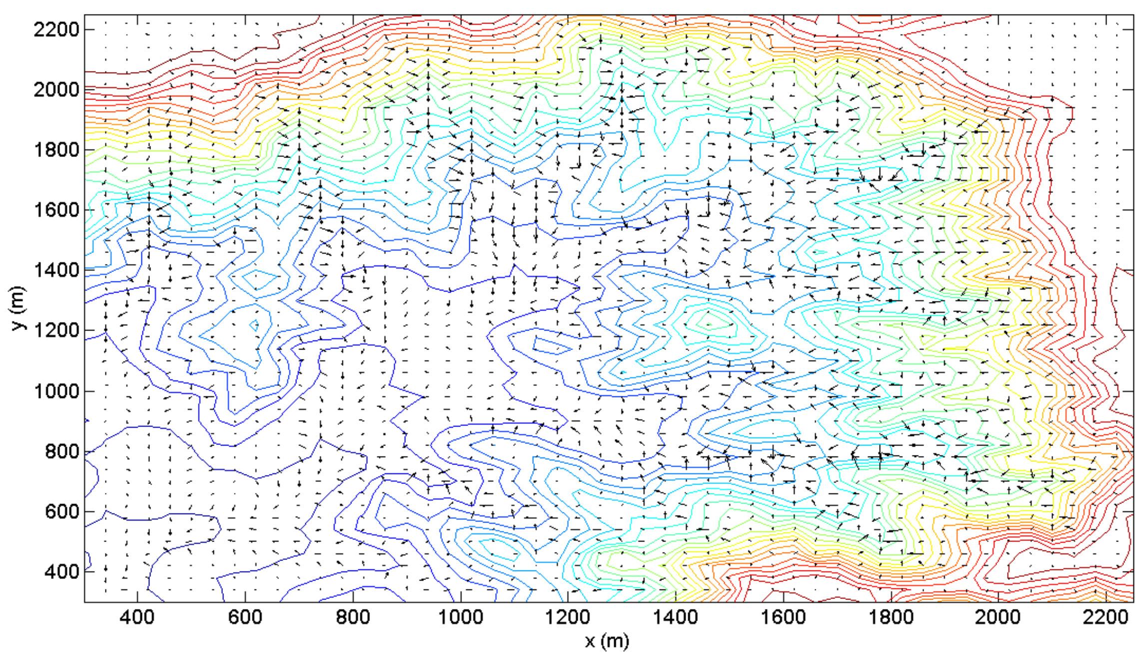

Figure 24. Test 3 – Flow path at t = 30 min

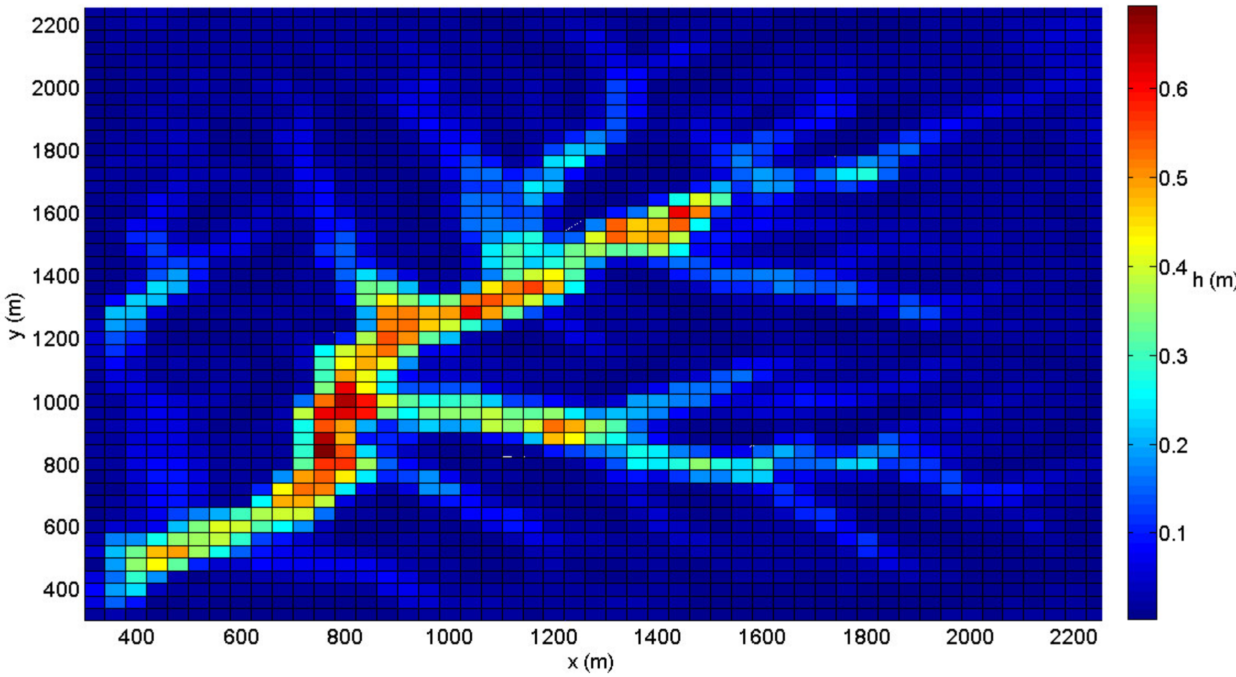

Figure 25. Test 3 – Water depths at t = 45 min

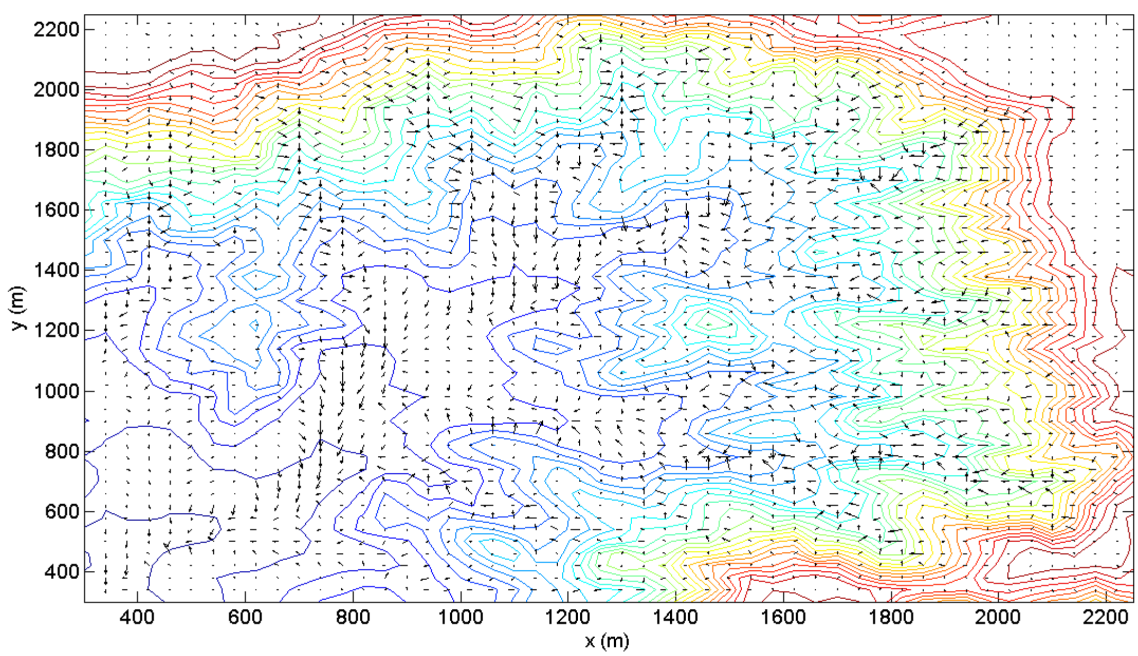

Figure 26. Test 3 – Flow path at t = 45 min

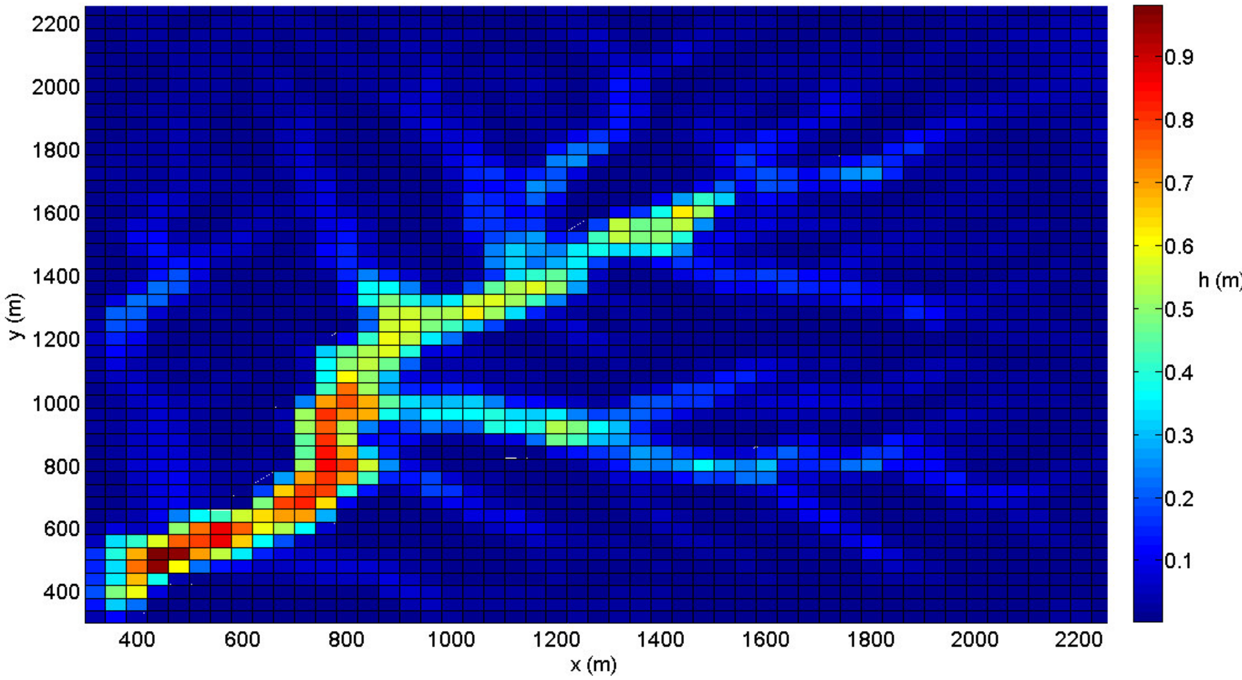

Figure 27. Test 3 – Water depths at t = 60 min

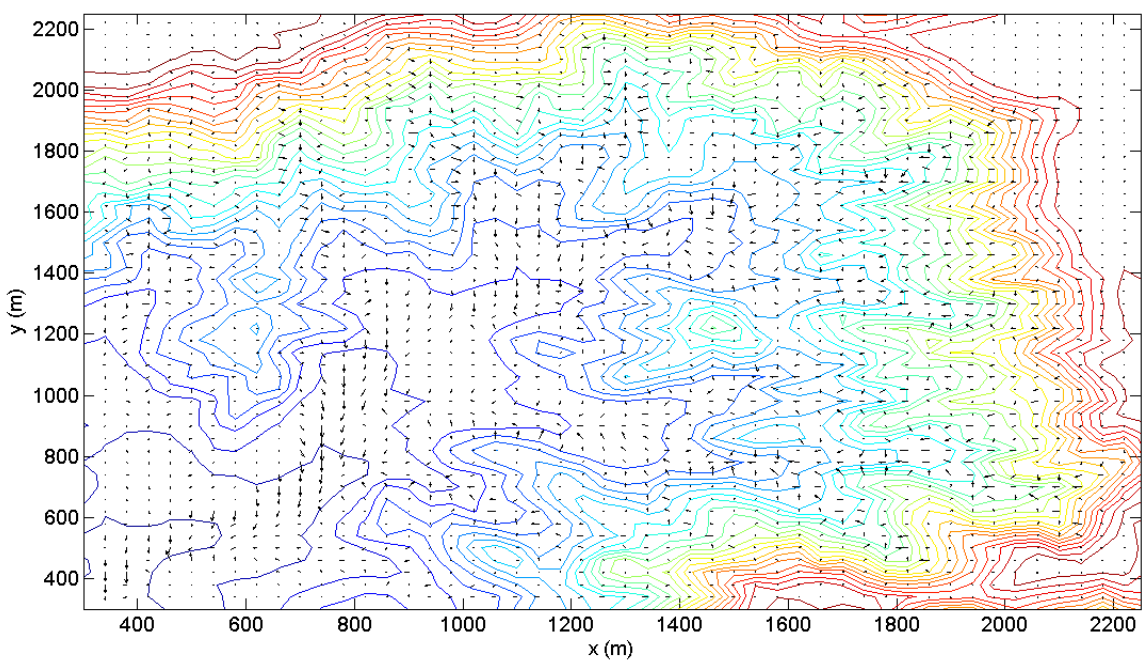

Figure 28. Test 3 – Flow path at t = 60 min

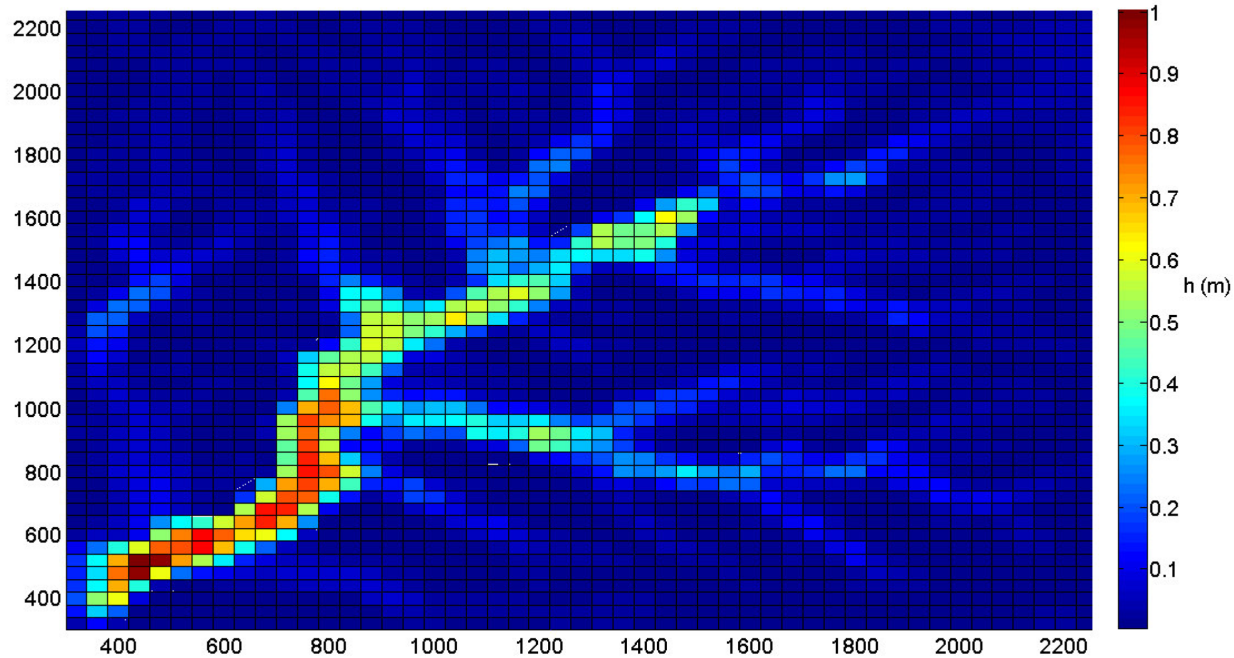

Figure 29. Test 3 – Water depths at *t* = 90 min

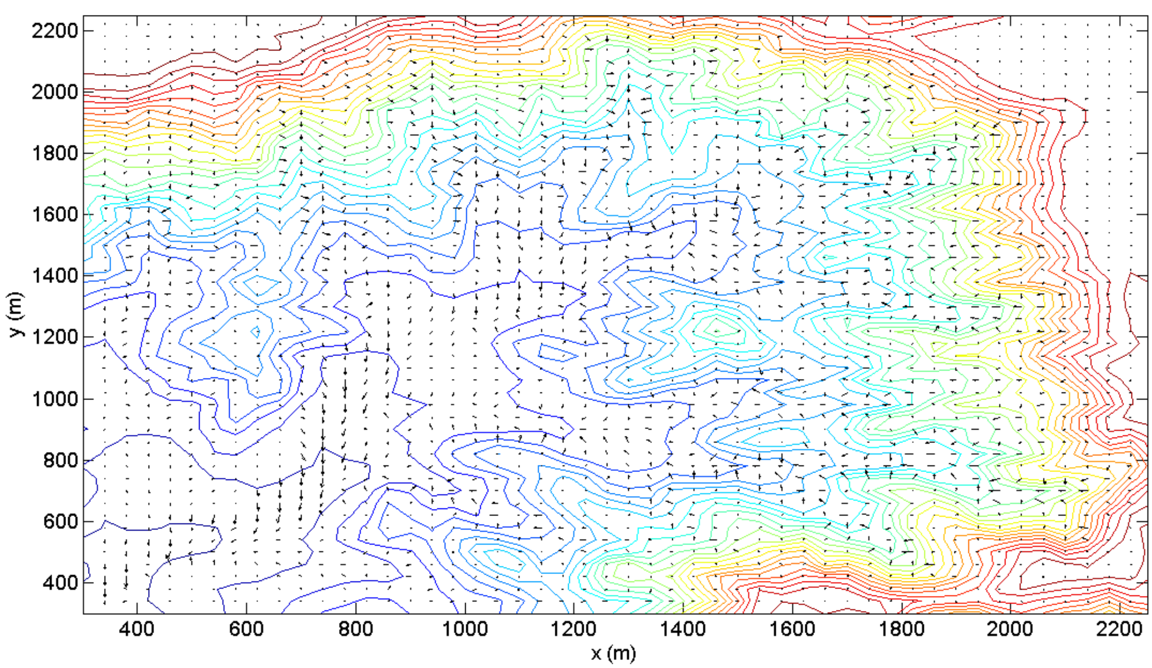

Figure 30. Test 3 – Flow path at *t* = 90 min

In figures 32 and 33 the relations between the friction coefficient and the discharge peak or the time to peak are shown. In particular, to physically evaluate the effectiveness of the implemented code, in figure 33 the numerical results in terms of time to peak have been compared with those obtained applying a formula of literature proposed by the Civil Engineering Department of the University of Maryland:

$$\tau = 26.3 \frac{(L/k_s)^{0.6}}{j^{0.4} i^{0.3}} \tag{34}$$

where:
τ is the time to peak;
L is the length of the drainage area;
K_s is the Strickler's coefficient;
j is the rainfall intensity;
i is the mean slope of the drainage area.

Moreover the discharge hydrographs have been compared with that obtained using the simple rainfall- runoff model (figure 34). Despite the rainfall- runoff model is independent by the choice of the Stickler's coefficient the results in terms of discharge peak, both the time to peak and the shape of the hydrographs are similar to those obtained by the implemented code.

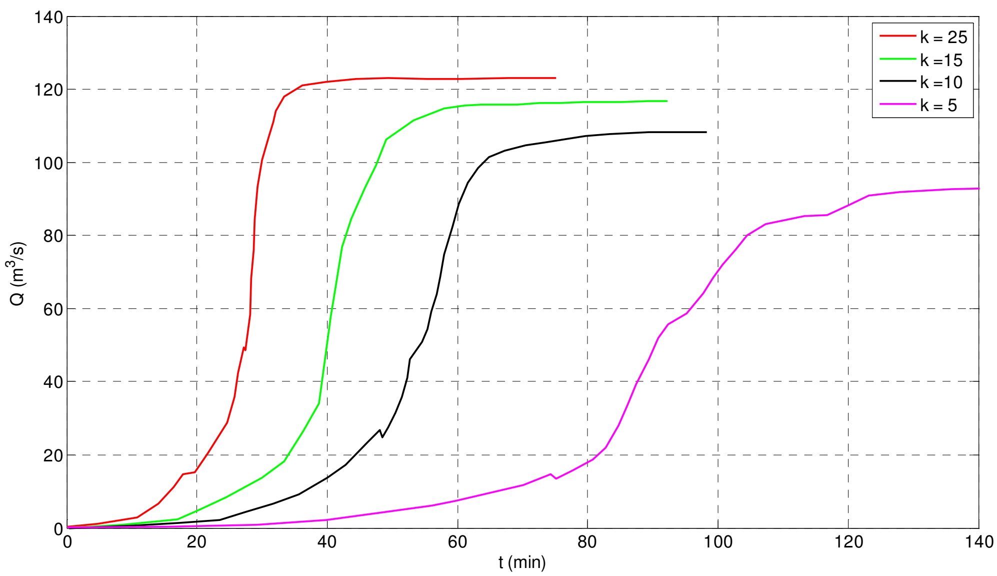

Figure 31. Test 3 – Discharge Hydrographs at the channel outlet computed with different Stickler's coefficients k (25, 15, 10, 5 $m^{1/3}/s$)

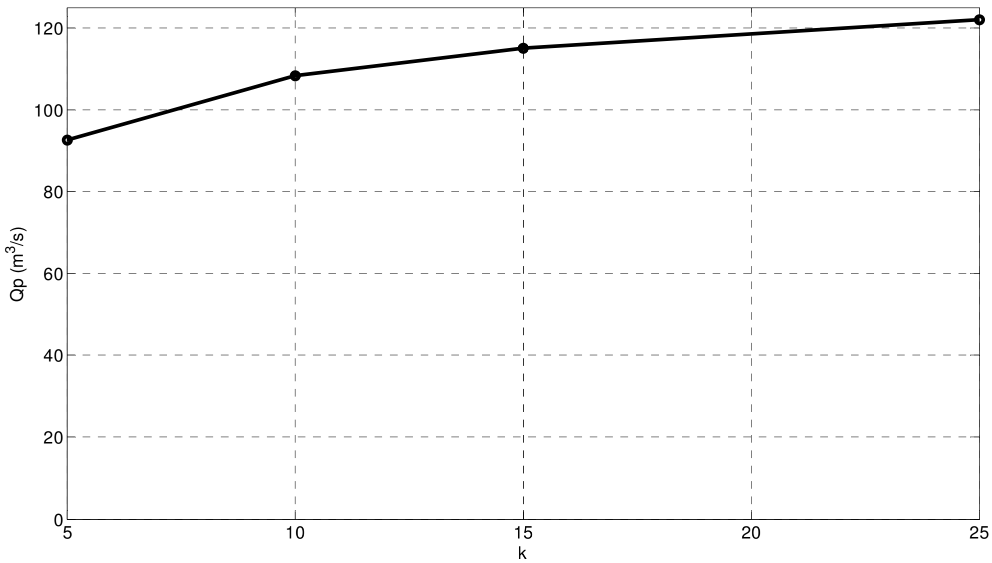

Figure 32. Test 3 – Relation between Strickler's coefficient k and discharge peak Q_p

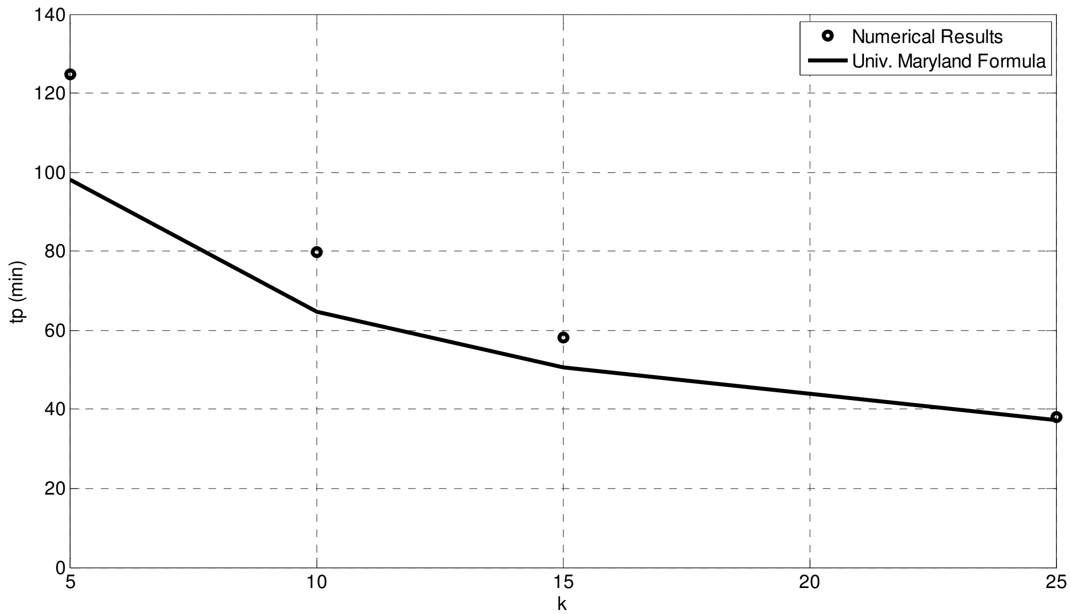

Figure 33. Test 3 – Relation between Strickler's coefficient k and time to peak t_p: comparison between computational results (**o**) and the University of Maryland predictive formula (**—**)

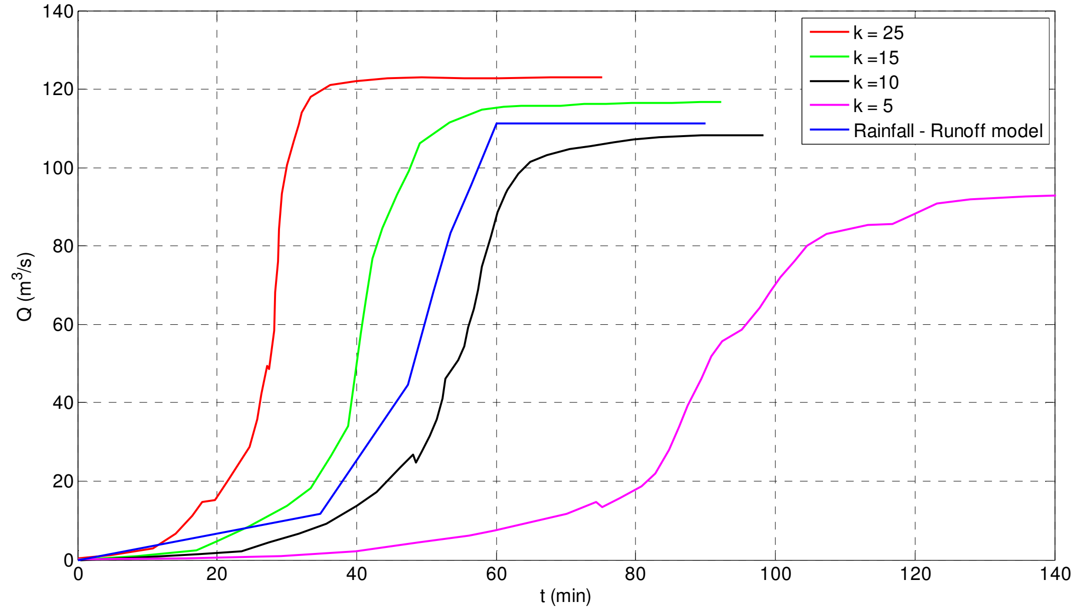

Figure 33. Test 3 – Comparison among the numerical discharge hydrographs and rainfall-runoff model

CONCLUSIONS

In the recent literature, there are a number of papers that propose the analysis of the overland flow problem by means of models applied to both ideal case and small real basin. Despite the large interest and the progress in rainfall–runoff modeling, there are very few papers concerning its application to simulate real large scale events. In this work a bibliographical review on some of these papers has been reported.

Then the second order in time and space MacCormack's scheme has been implemented in order to simulate real cases of overland flow. This has been a difficult task because of the presence of small water depths over high slope and irregular topography that may be induce numerical anomalies. Therefore in this work, to prevent these problems, some numerical techniques have been implemented. In particular the source term vector is decomposed in two different parts which are treated separately. The variation of the free surface is considered instead of the bottom slope. With regard to the friction term, a technique based on the limitation of the friction values has been implemented and, for small water depths, an implicit approach has been also used. Second order central schemes introduce numerical instabilities. To avoid such problem further terms are added to the shallow water equations according to the TVD theory. Moreover in this work, specific treatments for calculating wet/dry fronts have been also applied.

Finally the implemented code has been applied to simulate the surface runoff over three different real watersheds.

In the first two simulations the influence of the size of computational cell on the numerical results has been analyzed. The code provided stable results and quite similar peak values even using rather coarse grids.

Then the code was applied to the propagation of the surface runoff over a part of the Esaro Basin. The morphological description of the Esaro basin has been carried out by means of a GIS software. A constant rainfall intensity (100 mm/h) has been considered over the domain. For this case a sensitivity analysis of Strickler's coefficient has been made. The numerical results show that the choice of the Strickler's coefficient has a great influence on the peak discharge and on the time to peak. Further investigations will concern the numerical instability problems arising from overland flows due to very low rain intensities and in recessive limb of the flows. Moreover the burdensome computational time will have to be reduced.

References

Alcrudo, F., García Navarro, P. (1994). "Computing two dimensional flood propagation with a high resolution extension of McCormack's method". Proceedings of *Modelling of flood propagation over initially dry areas*, Molinaro, P. e Natale, L. (eds.), Milano, June 29 - July 1, 1994, ASCE: 3-17.

Aureli F., Mignosa P., Tomirotti M. (2000). "Numerical simulation and experimental verification of Dam-Break flows with shocks". *Journal of Hydraulic Research*, vol. 38 (3): 197-206

Ajayi A. E., van de Giesen N., Vlek P. (2008). "A numerical model for simulating Hortonian overland flow on tropical hillslopes with vegetation elements". *Hydrological Processes*, 22: 1107–1118.

Burguete J., Garcìa-Navarro P., Murillo J. (2008). "Friction term discretization and limitation to preserve stability and conservation in the 1D shallow-water model: Application to unsteady irrigation and river flow." *International Journal for Numerical Methods in Fluids* 58: 403–425

Caleffi V., Valiani A., Zanni A. (2003). "Finite volume method for simulating extreme flood events in natural channels". *Journal of Hydraulic Research*, vol. 41 (2): 167-177.

Fiedler, F. R., Ramirez, J. A. (2000). "A numerical method for simulating discontinuos shallow flow over an infiltrating surface". *International Journal of Numerical Method in Fluids*, 32: 219-240, John Wiley & Sons Ltd.

Costanzo, C., Macchione F. (2004). "Prestazioni di alcuni schemi bidimensionali ai volumi finiti per problemi di dam break". Proceedings of the *XXIX Convegno di Idraulica e Costruzioni Idrauliche*, Trento, 2004, Ed. Bios:Cosenza, 2004, 1: 829-836.

Costanzo, C., Macchione, F. (2005). "Comparison of two-dimensional finite volume schemes for dam break problem on an irregular geometry". Proceedings of the *XXXI IAHR Congress*, Seoul, Korea 2005, Theme D, 3372-3381. CD-ROM.

Costanzo C., Macchione F. (2006). "Tecniche numeriche per la simulazione della propagazione di piene su fondo irregolare". Proceedings of *Tecniche per la Difesa dall'Inquinamento*, G. Frega (a cura di), Editoriale Bios, Cosenza, 2006, Vol. 26 : 423-440.

Costanzo, C., Macchione F. (2006). "Modelli di simulazione dei fenomeni di allagamento in zona urbana: applicazione ad un caso reale". Proceedings of the *XXX Convegno di Idraulica e Costruzioni Idrauliche - IDRA2006*, Roma, 10 - 15 settembre, 2006, Casa Editrice Università degli Studi di Roma La Sapienza, CD-ROM.

England J. F. Jr., Velleux M. L., Julien P. Y. (2007). "Two-dimensional simulations of extreme floods on a large watershed". *Journal of Hydrology* 347, 229– 241

Esteves, M., Faucher, X., Galle, S., Vauclin, M. (2000). "Overland flow and infiltration modelling for small plots unsteady rain: numerical results versus observed values". *Journal of Hydrology*, Elsevier, 228: 265-282.

Fennema R. J., and Chaudhry M. (1990). "Explicit methods for 2-D transient free-surface flows". *Journal of Hydraulic Engineering*, Vol. 116 (8):

Fiedler F. R., Ramirez J. A. (2000). "A numerical method for simulating discontinuous shallow flow over an infiltrating surface". *International Journal for Numerical Methods in Fluids* (32) : 219–240

Gottardi, G., Venutelli, M. (2008). "An accurate time integration method for simplified overland flow models". *Advances in Water Resources*, Elsevier, 31: 173-180.

Harten, A., Hyman, J.M. (1983). "Self-adjusting grid methods for one-dimensional hyperbolic conservation laws". *Journal of Computational Physics*, 50: 235-269.

Hirsch C., 1990. *Numerical computation of internal and external flows*. John Wiley e Sons.

Howes D. A., Abrahams A. D., Pitman E. B. (2006). "One - and two-dimensional modelling of overland flow in semiarid shrubland, Jornada basin, New Mexico". *Hydrological Processes* 20, 1027–1046.

Jain M. K., Singh V. P. (2005). "DEM-based modelling of surface runoff using diffusion wave equation". *Journal of Hydrology* 302, 107–126

Jameson, A., Schmidt, W., and Turkel, E. (1981). "Numerical solutions of the Euler equations by finite volume methods using Runge-Kutta time-stepping schemes." *Proc. AIAA 14th Fluid and Plasma Dynamics Conference*, American Institute of Aeronautics and Astronautics, 81-1259.

Jinkang D., Shunping X., Youpeng X., Xu C., Singh V. P. (2007). "Development and testing of a simple physically-based distributed rainfall-runoff model for storm runoff simulation in humid forested basins". *Journal of Hydrology* 336, 334– 346

Kazezyılmaz-Alhan C. M., Medina M. A. Jr. (2007).”Kinematic and Diffusion Waves: Analytical and Numerical solutions to Overland and Channel Flow”. *Journal of Hydraulic Engineering*, Vol. 133, No. 2, 217 228

Liang, D., Binliang, L., Falconer, R.A. (2007). “Simulation of rapidly varying flow using an efficient TVD–MacCormack scheme”. *International Journal for Numerical Methods in Fluids* 53, 811–826.

Nujic, M. (1995). “Efficient implementation of non-oscillatory schemes for the computation of free-surface flows”. *Journal of Hydraulic Research*, 33(1), 101-111.

Shih D. S., Li M. H, Wu R. S. (2008). “Distributed Flood Simulations with Coupling Gauge Observations and Radar-rainfall Estimates”. *Water Resour Manage* 22:843–859

Taskinen A., Bruen M. (2007). “Incremental distributed modelling investigation in a small agricultural catchment: 1. Overland flow with comparison with the unit hydrograph model”. *Hydrological Processes* 21, 80–91

Venkata R. K., Eldho T. I., Rao E. P, Chithra N. R. (2008). “A Distributed Kinematic Wave–Philip Infiltration Watershed Model Using FEM, GIS and Remotely Sensed Data”. *Water Resour Manage* 22:737–755.

Yoon T. H., Kang S. K. (2004). “Finite volume model for two-dimensional shallow water flows on unstructured grids”. *Journal of Hydraulic Engineering*, ASCE, vol. 130 (7), pp. 678-688.